Chapter 10 – Circles

10.1 Lines and Segments that Intersect Circles
- 10.1.1 The "parts" of a circle: radius, diameter, chord, secant, tangent, circumference.
- 10.1.2 If a tangent line intersects a radius then it forms a 90° angle. ❶
- 10.1.3 Tangent segments to a common point are congruent. ❷

10.2 Arcs and Central Angles
- 10.2.1 Applying angle/segment addition postulate concepts to arcs.
- 10.2.2 Similarity between all circles

10.3 Chord (segment) Theorems
- 10.3.1 Congruent arcs ↔ congruent chords ❸
- 10.3.2 A diameter bisects a chord if and only if it is perpendicular to that chord ❹
- 10.3.3 Two chords are congruent if and only if they are equidistant from the center ❺

10.4 Inscribed Angles and Inscribed Polygons
- 10.4.1 Inscribed angles are half the arc they intercept. (intercept → form) ❻
- 10.4.2 Several corollaries from theorem 6.
- 10.4.3 The opposite angles of an inscribed quadrilateral are supplementary. ❼

10.5 Angle Theorems
- 10.5.1 If two segments intersect outside the circle then the angle is half the *difference* of the two arcs intersected. ❽
- 10.5.2 If two segments intersect inside the circle, then the angle is half the *sum* of the two arcs intersected. ❾
- 10.5.3 Corollaries of theorem 8 and 9

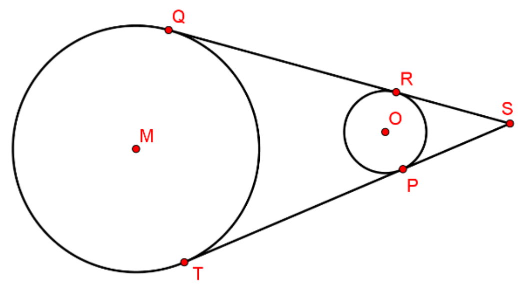
10.6 More Chord (segment) Theorems
- 10.6.1 If two chords intersect then for each chord the product of its parts is the same. ❿
- 10.6.2 If two secants/tangents intersect outside a circle then for each, the product of its exterior portion and entire length is the same. ⓫

10.7 Circles on the Coordinate Grid
- 10.7.1 Students must be able to both graph circles and write the equation for a circle that is already graphed. $(x-h)^2+(y-k)^2=r^2$. (Transformations/Pythagorean Theorem connections)
- 10.7.2 Using the Pythagorean Theorem to determine where a point lies in relation to a circle.

Practice Performance Task – 10.1
Nesting Proofs

The following performance task will guide you through writing a proof and then using that proof to complete a more complicated proof. Don't give up, there will be hints along the way.

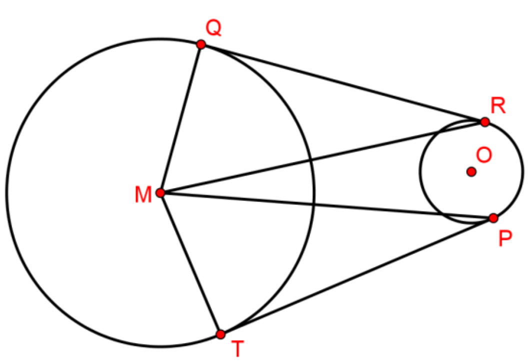
1. Theorem 2 states that tangent segments from a common point to a common circle must be congruent. Write which pairs of segments must be congruent based on this theorem. (Hint: there are two pairs.)



2. We now wish to prove $\overline{RQ} \cong \overline{PT}$. (You should NOT have this as one of your pairs for question 1.) Complete the right side of the given proof to show $\overline{RQ} \cong \overline{PT}$. Step 5 includes a hint.

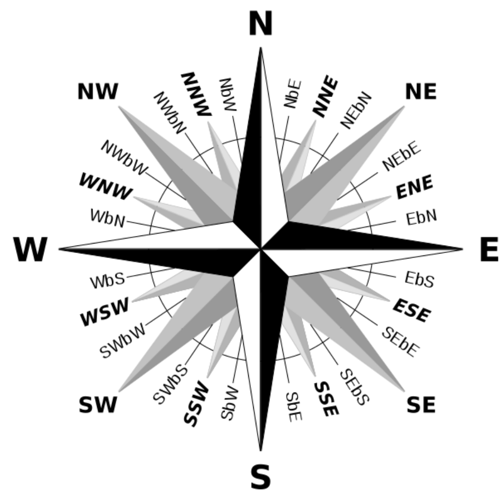
		Reasons
1	$SQ \cong ST$	
2	$SR \cong SP$	
3	$SQ = SR + RQ$	
4	$ST = SP + PT$	
5	$ST = SR + RQ$	*(3/1)
6	$SR + RQ = SP + PT$	
7	$SP + RQ = SP + PT$	
8	$RQ = PT$	

We have proven two tangent "sections" between two circles must be congruent. We'll call this the "Congruent Tangent Sections Theorem". You will use this in the proof below.



3. In the given diagram, we want to prove $\Delta MQR \cong \Delta MTP$. Explain how you can use the SAS theorem to prove the two triangles are congruent.

Practice Performance Task – 10.2
Compass Rose



The image to the right is a 32-point compass rose. There are four *cardinal* directions (North, East, South and West) and four *ordinal* directions (Northeast, Southeast, Southwest and Northwest). Together the cardinal and ordinal directions are called the *principal winds*.

1. How many degrees separates each principal wind?

2. How many degrees is the major arc connecting Southeast and West?

Bisecting each pair of adjacent principal winds is a *half-wind*. For example, between North and Northeast is North-Northeast (NNE).

3. How many degrees separates each half-wind from its closest principal wind?

4. A 90° arc is to be placed at random on the compass rose. What is the maximum number of half-winds it will cover? What minimum number?

The *quarter-winds* are formed by bisecting each adjacent direction previous discussed. For example, between Northwest (NW) and West-Northwest (WNW) is Northwest by West (NWbW). Confused yet?

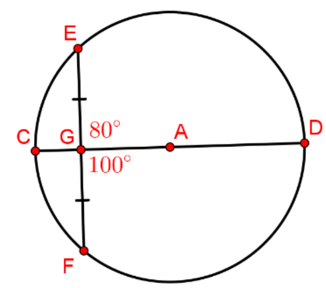
5. A ship traveling NE turns clockwise until it is traveling SEbE. How many degrees has the ship turned?

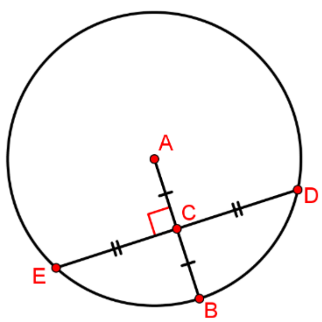
6. A ship traveling SWbW makes 78.75° CW turn. In what direction is it heading now?

Practice Performance Task – 10.3
Impossible! ... or is it?

Each of the following diagrams is either impossible (there is no way the diagram could be accurate), or is possible.

1. Determine if the following diagram is impossible or possible. In either case, you must use the theorems discussed in class to support your answer.





2. Determine if the following diagram is impossible or possible. In either case, you must use the theorems discussed in class to support your answer.

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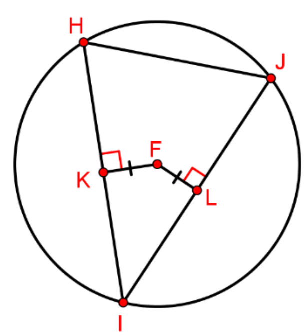
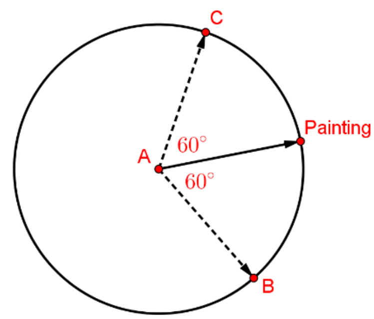


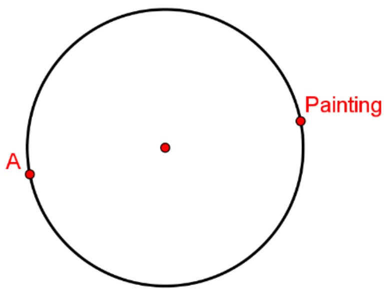
ΔHJI is an isosceles triangle.

Practice Performance Task – 10.4
Security Guards

The average human can see objects about 60° to the left of where they are looking and 60° to the right. Suppose a security guard is being paid to watch a famous painting being displayed in a circular room.

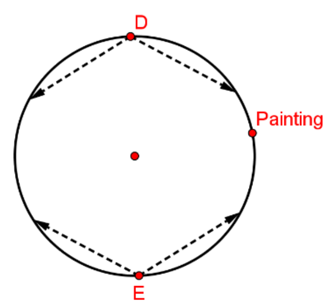
1. The security guard is standing in the center of the room and looking directly at the famous painting. How many degrees of the room (Arc CB) can the security guard see while looking directly at the painting? Explain.

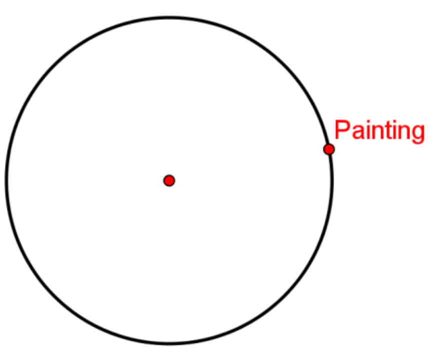




2. The security guard moves back to the edge of the room, opposite the famous painting. Assuming he has the same angle of vision and is looking directly at the painting, how many degrees of the room can he see? Complete the given diagram (like the diagram in part 1) to determine how many degrees of the room he can see.

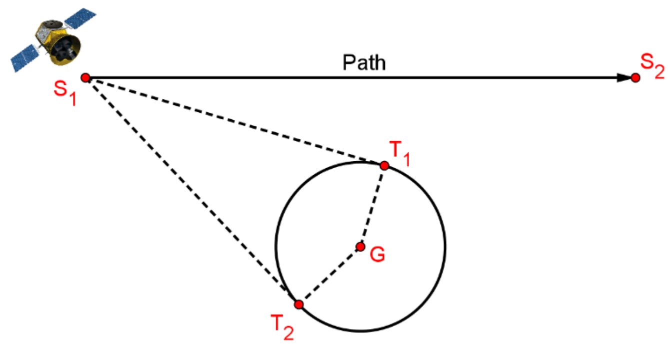
3. The gallery has decided to pay two security guards to stand at opposite sides of the gallery. We'll assume the guards are looking directly at each other (a bit awkward), and do not move their heads (even more awkward). How many degrees of the room can be viewed by both security guards?





4. How many security guards would the museum have to hire so that every part of the gallery can be seen by at least 2 guards? Use the diagram to the right to demonstrate how you would setup the guards.

Practice Performance Task – 10.5
Satellites



Suppose a satellite is flaying past Ganymede, the largest of Jupiter's 53 moons. S_1 is the earliest point at which the satellite can capture useful images of the moon. At an equal distance past the planet, S_2 is the last point at which the satellite can capture useful images.

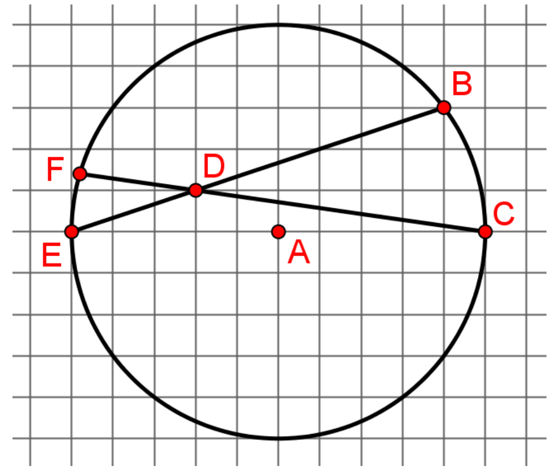
1. At S_1 the satellite can view 150° of Ganymede's surface (considering the 2D arc from T_1 to T_2). Find the measure of $\angle T_1S_1T_2$. Show your work or explain how you got the answer.

2. At S_1, the satellite is 250,000 miles from the center of Ganymede (G). Using the diagram above, draw $\overline{S_1G}$ and then find the measure of each of the angles of ΔS_1GT_2.

3. At S_1, find the distance from the satellite to each point of tangency (T_1 and T_2). Show all work.

4. As the satellite travels from S_1 to S_2, it takes images of the surface of the planet. At S_1 and again at S_2 it can view 150° of the moon's surface. 33° of the moon's surface is always visible to the satellite (Near the north pole). How many degrees of the moon's surface is never visible to the satellite?

Practice Performance Task – 10.6
Circles on the Coordinate Grid



A lattice point is any point on a coordinate grid where two grid lines cross. Another way to think about it is, a lattice point is any point with whole number x and y coordinates. In the diagram to the right, point F is the only point that is not on a lattice point. A is the center of the circle.

1. Let's consider A to be the origin of our graph (0,0). Use your ruler to draw the x and y-axis and then list the coordinates for each of the points that is a lattice point.

   B C

   D E

2. Use the distance formula to find the length of each of the following segments: Remember: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

   $\overline{BD}$

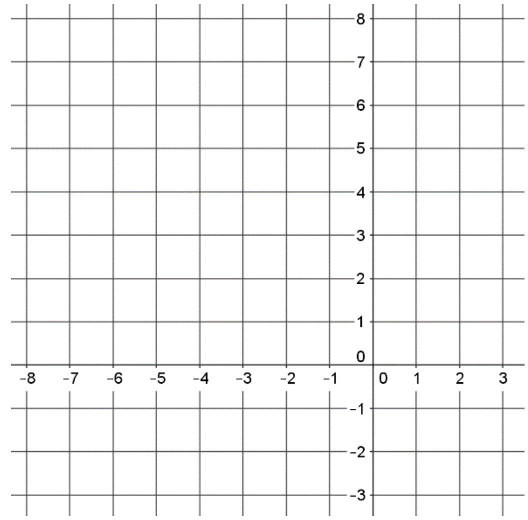
   $\overline{CD}$

   $\overline{DE}$

3. Find the length of $\overline{DF}$.

4. A fellow student claims D is the midpoint of $\overline{AF}$. Prove this is not true. Use complete sentences and show all mathematical evidence. Remember: $MP = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Practice Performance Task – 10.7
National Park Management



The grid to the right represents a large national park inhabited by several different animals. Use the grid to the right to complete the following questions.

1. Bears and wolves are common in this national park. The equation $(x+3)^2 + (y-3)^2 = 16$ represents the region populated by bears and $(x+1)^2 + (y-3)^2 = 4$ represents the area populated by wolves. Plot and label these two regions.

2. A backpacker goes missing from his campsite located at $(-7, -2)$. Since he was last seen, it is believed the backpacker could have traveled 4 miles in any direction. Write the equation for the circle that represents all his possible locations. Is the backpacker in risk of being attacked by a bear? A wolf?

3. Two years ago, a large forest fire burned within the park. It was centered at $(-5,3)$ and had a radius of 2 miles. Researchers wish to investigate the impact of this fire on areas populated by both bears and wolves. Name a potential coordinate location for researchers to begin their investigation. Explain why this is a good choice.

4. In a different region of the national park, the following set of equations describes the elevation change of the region. Use this information, without graphing, to answer the following questions.

Smoke has been seen rising from an area corresponding to $(5, -3)$. What is your best estimate for the elevation of that location? Show all necessary work.

Within the region…	…the elevation is @ or above…
$(x-3)^2 + (y+1)^2 = 29$	5000 feet
$(x-4)^2 + (y+1)^2 = 18$	6000 feet
$(x-4)^2 + (y+2)^2 = 9$	7000 feet
$(x-3)^2 + (y+2)^2 = 4$	8000 feet