Chapter 11 – Circumference, Area and Volume

11.1 Circumference and Arc Length
- 11.1.1 Converting from the measure of an arc (degrees) to its length (inches)
- 11.1.2 The relationship between circumference and “distance traveled” of a wheel/gear
- 11.1.3 Radians as an alternative to degrees (converting)

11.2 Areas of Circles and Sectors
- 11.2.1 Density as a measure of things/area
- 11.2.2 Area of a sector of a circle (using proportions)
- 11.2.3 Finding the area of an irregular shape (via addition/subtraction of areas)

11.3 Areas of polygons
- 11.3.1 Areas of rhombi and kites using diagonals
- 11.3.2 Areas of regular polygons ($A = \frac{1}{2} \cdot a \cdot p$)

11.4 Introduction to 3D figures (Combined with 11.5)
- 11.4.1 Classifying figures (naming)
- 11.4.2 Cross sections as “elevators” (Is the figure a polyhedron? Is it a prism?)
- 11.4.3 Polyhedron: polygonal faces. Prism: constant cross sections parallel to base

11.5 Volume of Prisms and Cylinders (Combined with 11.4)
- 11.5.1 $A_{prism} = B \cdot h$ (focus on identifying the base and determining its area)
- 11.5.2 Considering the cylinder as a pseudo-prism (our base is a circle)
- 11.5.3 Working with similar solids. (r, r^2, r^3)

11.6 Volume of Pyramids
- 11.6.1 Considering the traditional pyramid and pyramids with non-rectangular bases
- 11.6.2 Again, making a connection back to similarity

11.7 Volume of Cones
- 11.7.1 Same idea as the pyramid. Once again, we must take care to identify the correct height.
- 11.7.2 Surface area has been dropped from this section (covered in previous courses)

11.8 Surface Area and Volume of Spheres
- 11.8.1 For surface area, we consider the baseball as being made of 4 congruent circles
- 11.8.2 $V_{sphere} = \frac{4}{3} \cdot \pi \cdot r^3$
- 11.8.3 Make the historical connection to Archimedes’ discover that the volume of a sphere inscribed within a cylinder is $\frac{2}{3}$ the volume of the cylinder. It would take 1.5 baseballs to fill the cylinder.

Practice Performance Task – 11.1A
Unicycles and Gears

The following series of questions are connected to the following fact: *for every rotation of a circle, it travels a distance equal to its circumference.* Use this to answer the following series of questions.

1. A man is riding a unicycle. The unicycle's wheel has a diameter of 24 inches. The man rides the unicycle for 7 rotations of the wheel before he falls off. How far did he travel?

2. A child is also learning how to ride a unicycle. The child can also ride the unicycle for 7 rotations before he falls. The child's unicycle has a wheel with a diameter of 14 inches.

   a. How far can the child travel before falling?

   b. How much farther than the child can the man travel before falling?

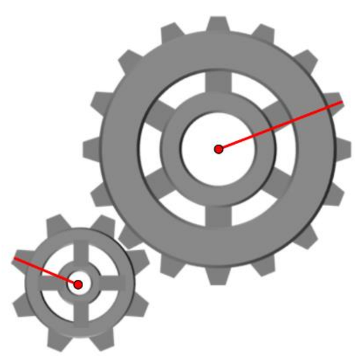
3. A bear is riding a unicycle. Her unicycle is even smaller than the child's; it has a diameter of 10 inches.

   a. How many rotations must the wheel of the bear's unicycle make to travel the same distance as ONE rotation of the boy's unicycle?

   b. How many rotations must the wheel of the bear's unicycle make to travel the same distance as ONE rotation of the man's unicycle?

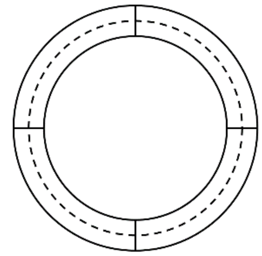
The following question-type is very common on the ASVAB (military aptitude test) and is very closely related to the above, silly, unicycle questions.

4. The smaller gear has a radius of 1.5 inches and the larger gear has a radius of 5.5 inches. How many rotations does the smaller gear complete during a single rotation of the larger gear?



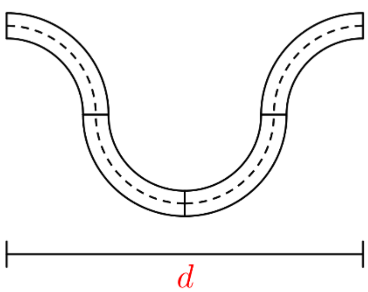
Practice Performance Task – 11.1B
Toy Race Track

Your younger sister received a toy racetrack for her birthday. Four pieces make a full circle, though they do not have to be arranged this way. When four pieces are used to make a circle, the circle has a radius of 10 inches.



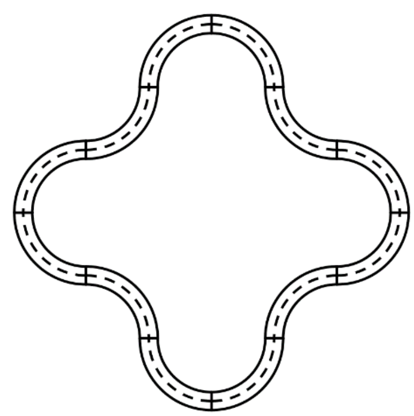
1. How many inches does the toy car travel each time it makes one trip around the track?

2. The box that the toy track and racecar came in claims the car can travel at a top speed of 100 feet per minute. How many trips around the circular track can the car make in 1 minute?



3. Your sister becomes bored with the circle and arranges the track as shown to the left. Find the horizontal distance traveled by the car (d).

4. The set of tracks came with 12 pieces of track. Your sister saw the kids in the commercial playing with a setup like the one shown to the right. It is your job as the older sibling to build this track. How many inches will the car travel each time it makes one trip around the track?

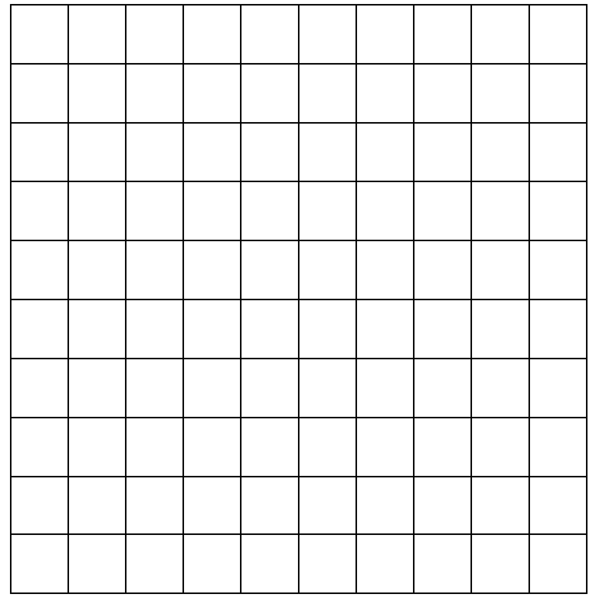


5. The wheels on the toy race care have diameters of .5 inches. How many rotations will each tire make each time it completes the track from problem 4?

Bonus: Find the area of the region contained by the track in problem 4.

Practice Performance Task – 11.2
Sprinklers

A landscaper is planning a sprinkler system to water a 10' x 10' square lawn. The grid to the right represents the lawn. For this problem, we will assume the sprinkler must be placed at a lattice point (where two gridlines cross). Each sprinkler can spray 5'.

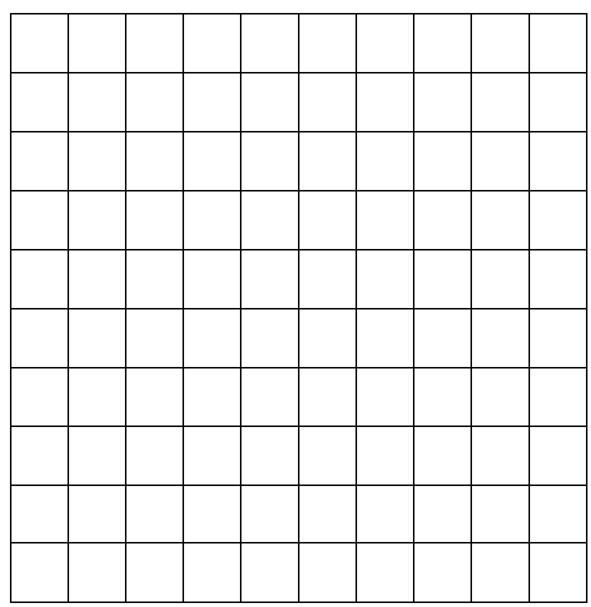


1. The landscaper's first plan is to place 1 sprinkler at each corner of the lawn. Use your compass and the provided grid to show the portion of the lawn that will be watered. Will the entire lawn be watered?

2. How many square feet of the lawn will be watered using the plan from part 1?

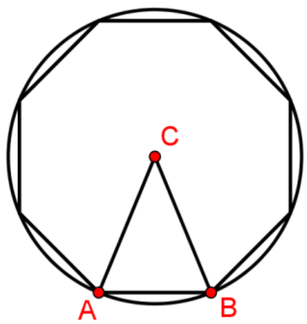
3. If the landscaper uses his first plan, how many square feet of grass will likely die because it did not receive water?

4. In addition to the sprinklers at the corners, a sprinkler is placed in the exact center of the lawn that sprays 5 feet of water 360°. Diagram this new plan using the grid given. How many square feet of the lawn is watered by two sprinklers?



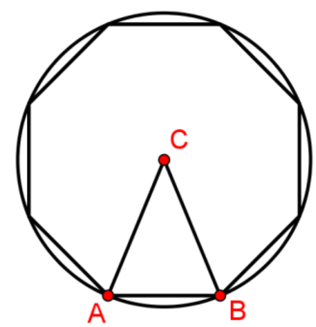
Practice Performance Task – 11.3
The Peruvian Sol

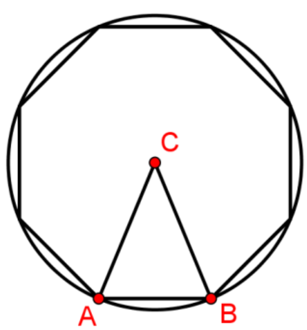
The Sol is the currency used in the nation of Peru. One Sol is worth about .31 US Dollars (31 cents). The image to the right is a 1 Sol coin. It is a circular coin with an inscribed regular octagon.



1. The diagram to the left represents the Peruvian 1 Sol coin. Find the measure of $\angle ACB$. Show all work.

2. The radius of the Peruvian 1 Sol coin is 12.75 mm. Use the diagram to the right to label the radius, draw the apothem and then calculate the length of the apothem. Show all work.





3. Determine the length of each side of the inscribed octagon. Use the diagram to the left to show all your work.

4. The formula $A = \frac{1}{2} \cdot a \cdot p$ can be used to find the area of a regular polygon. Calculate the area of the inscribed octagon.

Practice Performance Task – 11.4/11.5
Spinning Dollars

While many 3-dimensional figures can be formed by translating a cross-section, we can also form 3-dimensional figures by spinning 2-dimensional figures. In the following problem, we will consider the shape formed by spinning a dollar bill. To solve this problem, you will imagine the dollar bill to the right rotating around the line y to create a 3-dimensional shape.

1. Draw a sketch of the 3-dimensional figure formed by rotating the dollar bill around y. The US Dollar bill is 2.61 inches wide and 6.14 inches long. Label your sketch with its dimensions.

2. Calculate the volume of the object described in part 1. Show all work.

3. Suppose the dollar bill was spun around a vertical line that passes through the middle of the dollar bill, instead of y. Draw and label a sketch of the 3D figure created and calculate its volume.

4. Using the diagram to the left, draw any horizontal line. Imagine the dollar bill spun around the line you have created. Draw and label a sketch below and then calculate the volume of your shape.

Practice Performance Task – 11.6
The Pyramid of the Sun

The Pyramid of the sun is the largest pyramid in Teotihuacan, Mexico. The name of the pyramid comes from the Aztecs who discovered the abandoned city hundreds of years after the original people had abandoned it. Teotihuacan is located approximately 30 miles north of Mexico City.

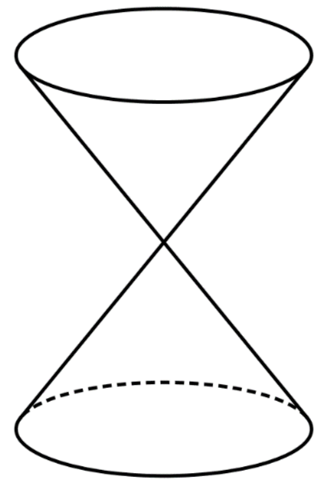
1. The Pyramid of the Sun is a square pyramid (ignore the top that looks a little flat) with 732 foot sides and a height of 233 feet. Draw a diagram of the pyramid and label it with the given dimensions.

2. Calculate the volume of The Pyramid of the Sun.

3. Tourists can climb to the top of the pyramid. How many feet is the walk up the side of the pyramid? (Assuming you take the shortest path up the middle of one face.)

4. The faces of the pyramid are steep, an incline of about 32°. Suppose you hike up the pyramid at a pace of about 1 mile per hour. How many minutes would it take to reach the top? (1 mile = 5280 feet and 1 hour = 60 minutes)

Practice Performance Task – 11.7
The Hourglass

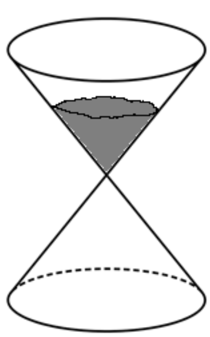


The figure to the right shows a simplified hourglass made of two right-cones with a common vertex. The top cone is full of sand and slowly drains into the bottom cone. The amount of time it takes for the top cone to completely empty is always the same.

1. The hourglass is 15 cm tall with a radius of 5 cm. How many cubic centimeters of sand are in the hourglass? Show all work.

2. The sand drains from the top cone at a rate of 9.8 cubic centimeters per minute. How long will it take for all the sand to drain from the top of the cone?

3. The hourglass is designed so the owner can remove or add sand to change the amount of time the hourglass measures. If the hourglass begins with the top cone entirely full and the bottom cone empty, how many cubic centimeters of sand must be removed so the hourglass takes 15 minutes to drain?



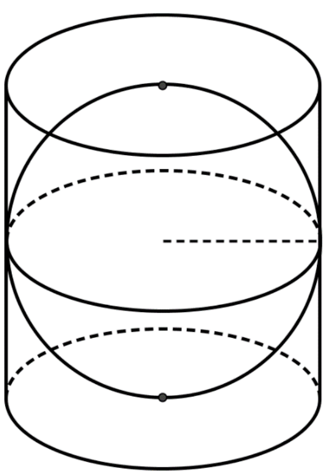
4. Instead of the top cone being full, you pour half the sand out. Describe the height and radius of the cone of sand held in the top cone before it begins to drain. The gray cone in the diagram. (Hint: it doesn't have a radius of 2.5 cm and a height of 3.75 cm. Sorry, not that easy.)

Practice Performance Task – 11.8
Memerobilia

A tradition for many young baseball fans is to have their favorite player sign a baseball. If your favorite baseball player is good then a signed baseball can be worth a lot of money. To solve the following problem, assume baseballs have a radius of 3.8 cm.

1. Calculate the volume (in cubic centimeters) of a baseball.

2. One style of display case is a clear plastic cube. The case is sized so that the baseball touches each of its faces. What are the dimensions of the cube case? Calculate its volume.



3. Some cases are cylindrical instead of cubic. The diagram to the left shows a case that is a cylinder. Once again, the baseball touches the top, bottom and side of the case. Label the dimensions of the case on the diagram and then calculate the volume of the case.

4. Which of the two cases fits the baseball more snugly? In other words, which case has the least amount of empty space in the case around the baseball? Show all work.