

Chapter 2 – Reasoning and Proofs

2.1 Conditional Statements

- 2.1.1 The family of conditionals: converse, inverse, contrapositive
- 2.1.2 Definitions and Biconditional statements
- 2.1.3 Create and interpret truth tables

2.2 Inductive and Deductive Reasoning

- 2.2.1 Conjecturing using pattern recognition (Inductive Reasoning)
- 2.2.2 Using facts and definitions to prove a conjecture (Deductive Reasoning)
- 2.2.3 Logic patterns such as syllogism and detachment

2.3 Postulates and Diagrams

- 2.3.1 Introduction and illustration of a series of postulates involving points, lines and planes

2.4 Algebraic Reasoning

- 2.4.1 Write two-column proofs using the equality properties
- 2.4.2 Reflexive, symmetric, and transitive properties

2.5 Proving Statements about Segments and Angles

- 2.5.1 Write two-column proofs using the geometric postulates learned in chapter 1

2.6 Proving Geometric Relationships

- 2.6.1 Alternative proof formats: flow proofs and paragraph proofs
- 2.6.2 Prove a variety of theorems: Congruent supplements, vertical, linear pairs, etc.
- 2.6.3 Combine geometric and algebraic reasoning in proofs

Practice Performance Task – 2.1
Flamingos



A middle school student wrote the following passage about three different species of flamingos. Read the passage and use it to complete the questions that follow.

While there are six different species of flamingos, the three most common are the American Flamingo, Greater Flamingo, and the Lesser Flamingo. The American Flamingo is most commonly found in the Caribbean. It is the only Flamingo that naturally lives in North America. The life expectancy of the American Flamingo in the wild is 40 years. The Greater Flamingo is the largest of the three main species. The Greater flamingo tends to be a lighter shade of pink than the other species. The Lesser Flamingo is the smallest of the main species. There are more Lesser Flamingos in the world than any other species. The Lesser Flamingo is about three feet tall when full grown. Lesser Flamingos are very common in zoos because they adapt well to captivity.

1. Choose one sentence from the passage above and copy it onto line #1. (You cannot choose the underlined sentence.) On line #2 write the sentence as a conditional statement using the if/then format.

1. _____

2. _____

2. Is the statement you wrote on line #2, biconditional? Make sure to explain your reasoning.

3. The original passage has an underlined sentence. Below, the underlined sentence has been written in four forms: $p \rightarrow q$ (original), $q \rightarrow p$ (converse), $\sim p \rightarrow \sim q$ (inverse), $\sim q \rightarrow \sim p$ (contrapositive). Correctly identify which is which and then determine if the conditional statement must be true or false. Assume the original conditional is true.

_____ (T/F) If the flamingo is a Lesser Flamingo, then it is about three feet tall when full grown.

_____ (T/F) If the flamingo is not a Lesser Flamingo then it is not about three feet tall when full grown.

_____ (T/F) If the flamingo is not about three feet tall when full grown then it is not a Lesser Flamingo.

_____ (T/F) If the flamingo is about three feet tall when full grown, then it is a Lesser Flamingo.

Practice Performance Task – 2.2
Riddles and Puzzles

Read the following riddle and answer question #1.

A spy wants to enter a castle, but a guard protects the door. The spy must know the password to enter. In order to overhear other people say the password, the spy waits in a nearby bush. A baker approaches the guard, and the guard says "If I say 12, what do you reply?" The baker replies "6". "You may enter." A few minutes later, a blacksmith approaches and the guard asks "If I say 6, what do you reply?" The blacksmith replies, "3". "You may enter." The spy puts on a disguise and approaches the guard. "If I say 4, what do you reply?" The spy replies "2". The guard immediately arrests the spy.

1. The above story involves several elements of logic discussed in class. Using complete sentences, describe the story in terms of the logic employed. You do NOT need to solve the riddle. Your description must include at least 3 of the following terms: deductive reasoning, inductive reasoning, law of detachment, law of syllogism, conjecture, counter example.

Bonus: What should the spy have said, why?

Lewis Carroll is famous for writing Alice in Wonderland, but was a professor of mathematics specializing in logic. Below, one of his famous puzzles is both posed and partially solved.

Assume the following facts are true: (a) All babies are illogical, (b) Nobody is hated who can wrestle a crocodile, (c) Illogical people are hated. Prove that no babies can wrestle a crocodile.



- | | |
|--|----------------------------|
| 1. If it is a baby, then it is illogical | Given (a) |
| 2. If it is illogical, then it is hated | Given (c) |
| 3. | Law of Syllogism #1 and #2 |
| 4. If it can wrestle a crocodile, then it is not hated | Given |
| 5. | Contrapositive of #4 |
| 6. | Law of Syllogism #3 and #5 |

2. Lines 3, 5 and 6 of the above solutions have been left blank. Complete the missing lines below.

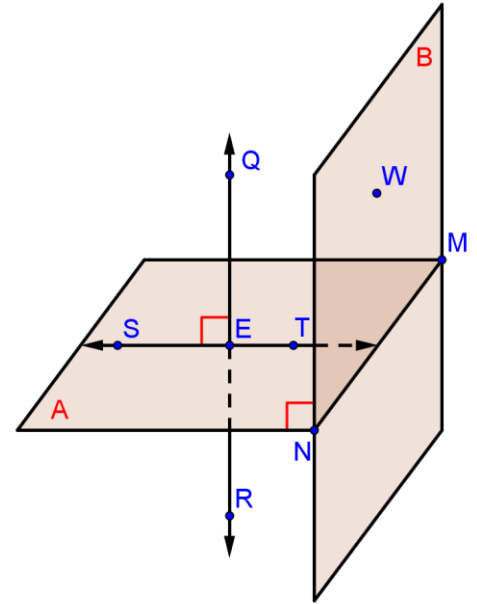
#3 _____

#5 _____

#6 _____

Practice Performance Task – 2.3
Geometric Diagrams

The diagram to the right was placed on a high school geometry exam. Students were to write three true statements and explain how they know they are true. Three answers are shown below. For each statement, explain whether or not the statement and evidence are correct. In some cases, the necessary evidence may be missing; you must provide the evidence if possible.



1. According to the Three Point Postulate, plane A is the only possible plane that passes through points S, E and T.
2. Line QR is perpendicular to line ST and plane A is perpendicular to plane B.
3. Points S, E, T and W are coplanar even though the plane is not pictured in the diagram.

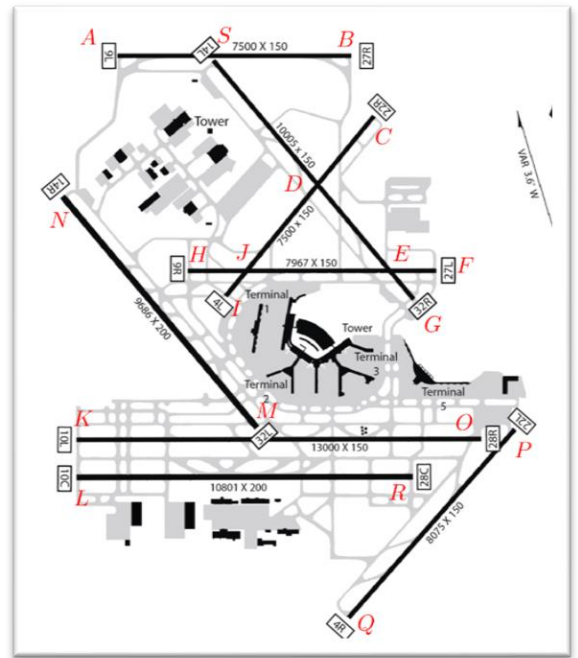
Use the following description of a diagram to answer question 4 and 5.

\overrightarrow{AB} and \overrightarrow{CD} intersect at a right angle at point E. Point A, E and B are lie on plane J but point C and D do not. Finally, construct \overrightarrow{AD} .

4. Use the space provided to the right of this question to construct a diagram that matches the given description. Remember, there multiple ways to draw your diagram. If you need more room you may use the back of this paper.
5. Is \overrightarrow{AD} contained within plane J in your diagram? Is it possible to draw the diagram in such a way that plane J would contain \overrightarrow{AD} ? Explain your reasoning.

Practice Performance Task – 2.5
Airports

The diagram to the right shows the major runways at Chicago O’Hare International Airport. O’Hare is the second busiest airport in the US as measured by number of departures. Use the diagram to the right to answer the questions that follow.



- Suppose we know D is the midpoint of \overline{SG} and $\overline{DG} \cong \overline{DI}$. Complete the following proof to show $\overline{SD} \cong \overline{DI}$.

Statements	Reasons
D is the midpoint of \overline{SG}	
	Definition of a midpoint
$\overline{DG} \cong \overline{DI}$	

- Suppose we know $\angle HJD \cong \angle DEF$ and $\angle HJD \cong \angle ASD$. Complete the following table to prove $\angle DEJ \cong \angle BSD$.

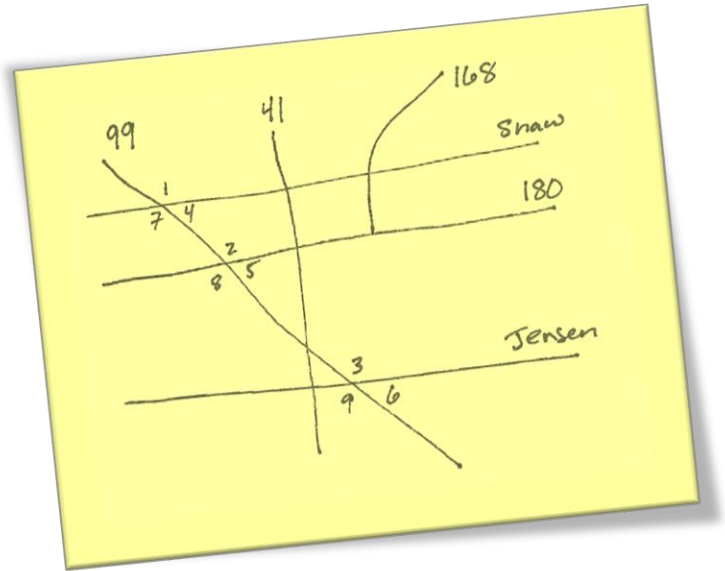
Statements	Reasons

- Consider the following argument posed by a high school student. Determine if the student is correct and explain why.

The problem tells us that D is the midpoint of \overline{SG} . That means line \overline{CJ} cuts \overline{SG} in half and \overline{SG} cuts \overline{CJ} in half. Since both segments are cut in half, all the halves must be equal. Since each \overline{SG} and \overline{CJ} each have two halves, the two segments must be congruent.

Practice Performance Task – 2.6
Map Sketches

In Vermont, where I'm from, there is only one major freeway. When I moved to California, I was confused by the four major freeways that pass through Fresno. I drew a basic map, with a few extra surface streets, on a Post-it, and kept it in my wallet so I would not get lost. Use the Post-it to the right to answer the questions that follow.



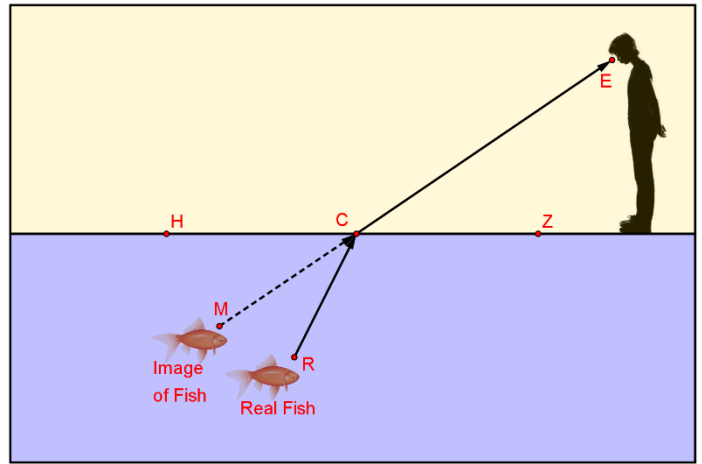
- Suppose we know $\angle 7 \cong \angle 8$. Complete a flow proof to show $\angle 1 \cong \angle 2$.

- Suppose we know $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. In addition, we know $m\angle 1 = (14x + 42)^\circ$ and $m\angle 6 = (6x - 2)^\circ$. Complete the two column table below to first prove $\angle 1$ and $\angle 6$ are supplementary and then find the measure of each angle.

Statement	Reason
$\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$	
	Transitive property equality
$\angle 3$ and $\angle 6$ form a linear pair of angles	
	Linear pair postulate
	Definition of supplementary angles
$m\angle 1 + m\angle 6 = 180^\circ$	
$m\angle 1 = (14x + 42)^\circ$ and $m\angle 6 = (6x - 2)^\circ$	
	Substitution Property of Equality

Practice Performance Task – 2.6
Light Refraction

When light travels from one medium (like air) to another medium (like water) it is refracted (changes directions). For example, if you were standing next to a lake looking at a fish in the water, the real fish (point R) is actually deeper than the image you see (point I). If you were attempting to shoot the fish with an arrow, you would have to aim below the image you see in the water to hit the real fish!



Use this information and the diagram to the right to answer the questions that follow.

- Suppose you are working on your homework with a friend who is also in geometry. He makes the following argument about the diagram:

I know angle EOQ and IOR are vertical angles because they make an "X". In my notes I see the vertical Angles Congruence Theorem says those two angles have to be congruent because they are vertical.

In complete sentences, explain what mistakes your friend has made in interpreting the diagram. Explain why this mistake is so common among high school students.

- In the diagram to the right, a line perpendicular to the surface to the water has been added. Explain how we know :
 $\angle HOI + \angle UOE = 90^\circ$.

