Chapter 4 – Transformations

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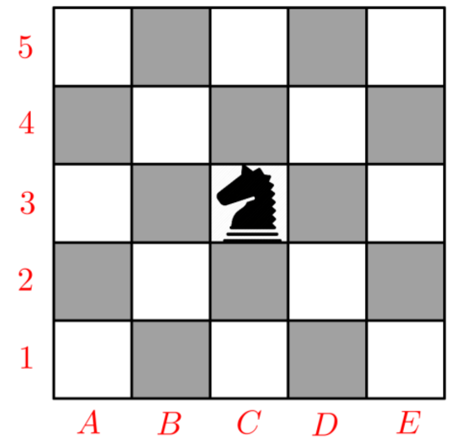
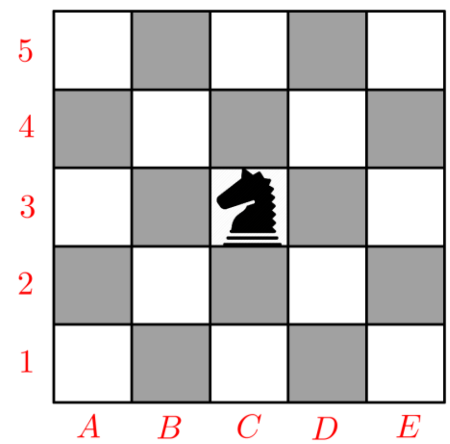
Practice Performance Task – 4.1
The Knight's Tour

The Knight's Tour is a problem which requires a chess piece to be moved to each square on a board without touching the same square twice. The problem is well-known in a field of mathematics called graph theory as well as in computer science where students write programs to solve the problem. The following problem will introduce the basics of The Knight's Tour.

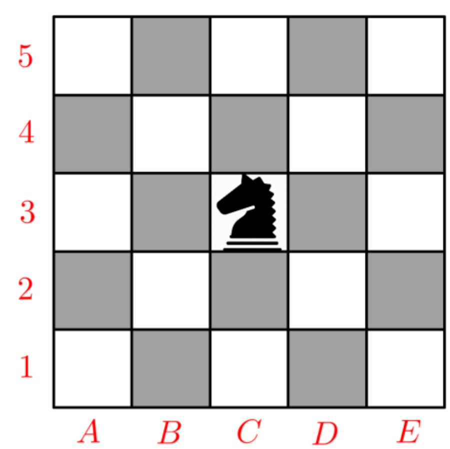
In chess, the knight (the one that looks like a horse) moves in an "L" shape which can be described by the vectors $< \pm 1, \pm 2 >$ and $< \pm 2, \pm 1 >$. For example, one valid move would be $< -1,2 >$ which would move the knight from C3 to B5. Another valid move would follow the vector $< -2, -1 >$.



1. Once again, assume the knight begins on the C3 square. Write the translation rule (function notation) that matches the translation vector: $< 2,1 >$. Afterwards, place the letter "N" on the image of the knight after the translation.



2. Remember, the goal of this problem is to move the knight from one square to another until all the squares have been touched. You cannot touch the same square twice. One solution to The Knight's Tour begins with the translations: $(x, y) \rightarrow (x + 2, y + 1)$, $(x, y) \rightarrow (x - 2, y + 1)$, $(x, y) \rightarrow (x - 2, y - 1)$. Remember, the knigh begins on C3 and each translation begins where the previous translation moved the knight. Use the diagram to the right to map the first three moves of this solution and then compose the three transformations into one transformation.



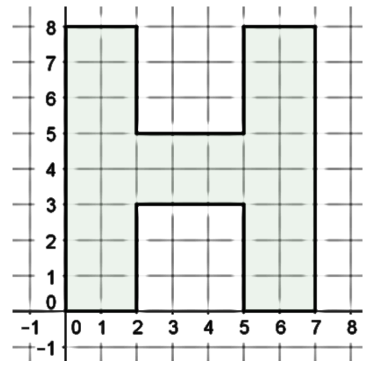
3. Your friend tells you that she thinks she has the first 5 moves worked out. She gives you her solution using vectors. Are her first 5 moves valid according to the rules of the problem? Explain.

   Moves: $< -1, -2 >; < 2,1 >; < -1,2 >; < 2,1 >; < -1, -2 >$

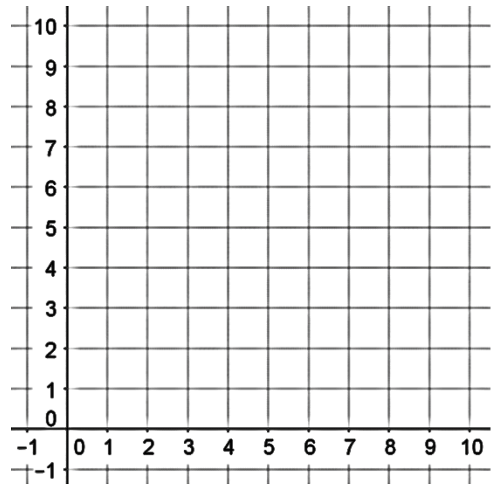
Practice Performance Task – 4.2
Alphabet Reflections

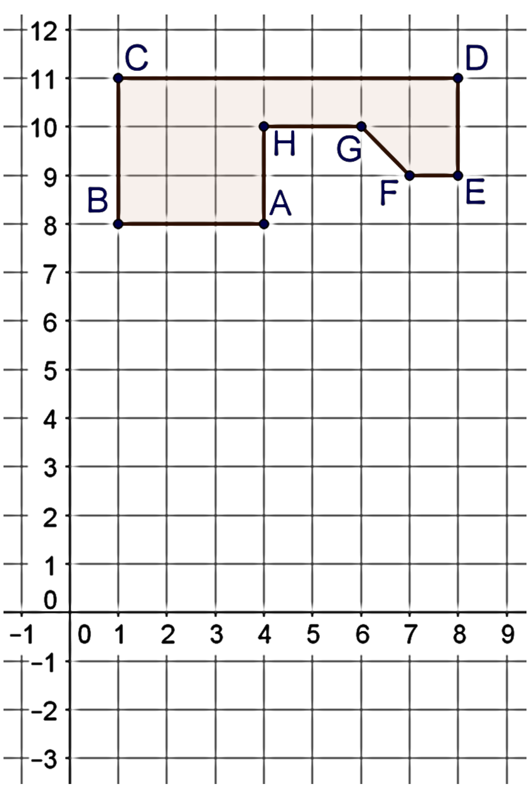
Several letters in the English alphabet exhibit reflectional symmetry. The problem that follows will investigate the types of reflectional symmetry present within the English alphabet.

1. The letter "H" is shown plotted on a coordinate plane. Write the equation for and plot each line of reflection.



2. Use the coordinate plane to the right to plot a letter in block style (similar to the "H" in the previous problem). The letter you choose must exhibit reflectional symmetry with **only one** line of reflection. After plotting the letter, plot the line and write the equation below.





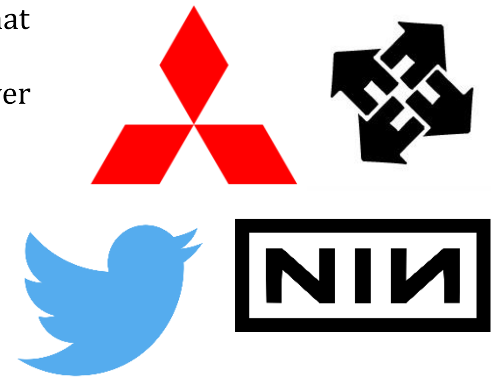
3. The figure to the left must be reflected multiple times to form a letter from the English alphabet. Follow the given directions to complete the letter. All lines of reflection must be drawn on the diagram. What letter is created?

First, reflect the figure over the line $y=8$. We will call the image after this reflection part 2. Now, take part 2 and reflect it over the line $y=5$. Finally, take part 3 and reflect it over the line $y=2$. When done, you will have your letter.

Practice Performance Task – 4.3
Logos

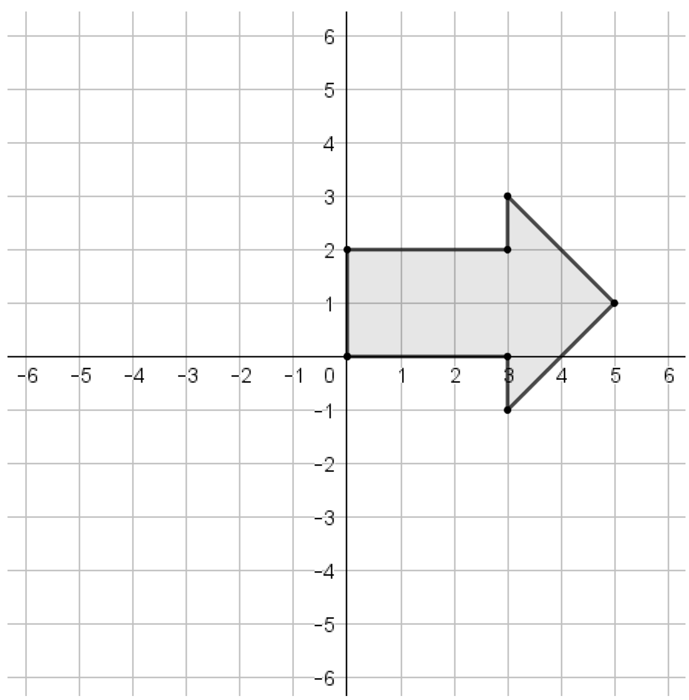
Many business and team logos exhibit reflectional and rotational symmetry. Logos that are symmetric may draw attention, be more memorable or simply appear more appealing to look at. In the following problems we will consider several famous logos that employ symmetry.

1. To the right there are several famous logos. Choose one logo that exhibits rotational symmetry but does NOT use reflectional symmetry. Identify the center and angle of rotation. Your answer should include a rough sketch of the figure.



2. Using the same set of logos from problem one. Choose a logo that exhibits BOTH reflectional symmetry and rotational symmetry. Identify all lines of reflection as well as the center and angle of rotation. Your answer should include a rough sketch of the figure.

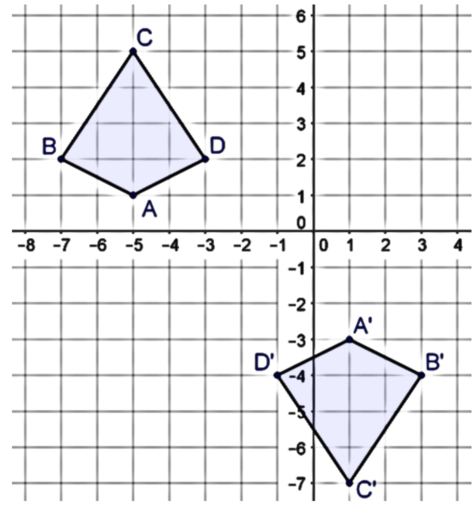
3. Suppose you work in advertising for a delivery company. You are working on a new logo that will have 90° rotational symmetry about the origin. The pre-image for the logo is plotted. Complete the logo.



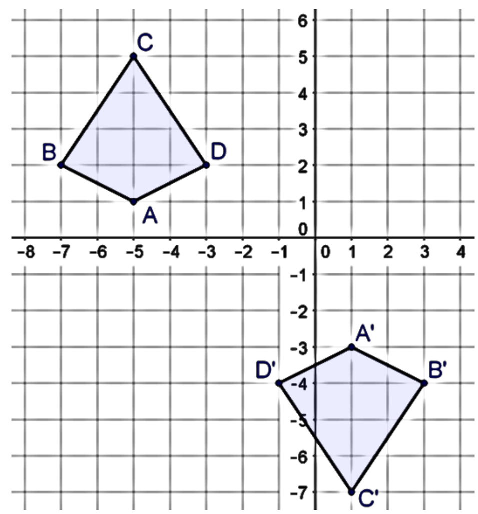
Practice Performance Task – 4.4
Alternate Transformations

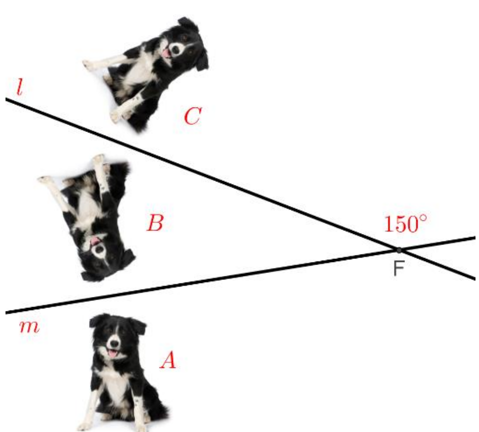
Sometimes there are multiple ways to compose transformations and still end at the same image. In some cases, there may be a shorter series of transformations possible. In the following set of problems, you will need to be creative as you think about multiple sequences of transformations.

1. The diagram to the right shows the preimage $ABCD$ and the image $A'B'C'D'$. Describe the congruence transformations that map $ABCD$ onto $A'B'C'D'$ using only reflections. You must write out the steps and draw any intermediate images on the diagram.



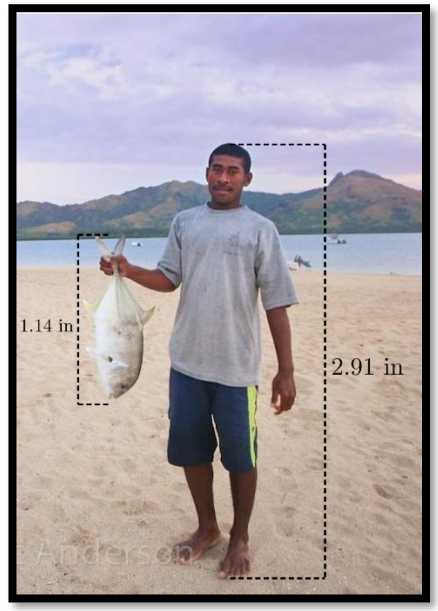
2. The same diagram from problem one has been copied to the right. Describe a different set of congruence transformations to map $ABCD$ onto $A'B'C'D'$ using only translations and rotations. You must write out the steps and draw any intermediate images on the diagram.





3. To the right we have three images of border collies. A is the pre-image, B is the first image, and C is the second image. Describe two sequences of transformations that map A onto C. Include all the necessary details.

Practice Performance Task – 4.5
Photographs and Trees



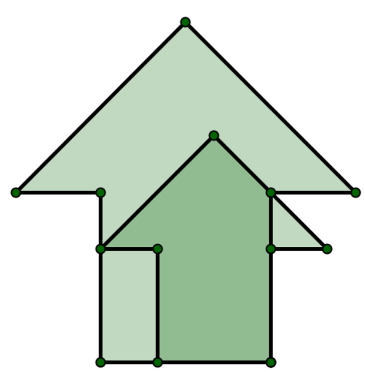
Your friend was in Mexico over the summer break with his family. When he returned, he began bragging about the size of the fish he caught. The picture to the right shows your friend holding the largest fish he caught.

1. Your friend is 5 feet 8 inches tall. What is the scale factor between this photograph of your friend and your actual friend? Remember, there are 12 inches in 1 foot. Show all work.

2. Your friend claims the fish was over 2 feet long. Is your friend being honest with you? How long was the actual fish if it is 1.14 in. in the picture? Show all work.

3. If the fish had been 18 inches when your friend caught it, then how long would it appear to be in the photograph?

4. The logo to the right is being considered for a new forest conservation organization. Describe the new logo in terms of the dilations used. Your description should discuss the center and approximate scale factor.



Practice Performance Task – 4.6
Expanding Ideas

According to the textbook: "Two geometric figures are similar figures if and only if there is a similarity transformation that maps one of the figures onto the other." With this definition in mind, the following problems will attempt to apply similarity to objects we have not yet discussed.

1. Teachers are sometimes guilty of saying things without properly explaining them. In class one day, your teacher says, "*Obviously all circles are similar to each other*." Using the definition of similarity given above, explain why this statement is true. Use complete sentences and diagrams if necessary.

2. While you are still trying to wrap your mind around the statement about circles, a fellow classmate raises his hand and asks: "*Wait, if all circles are similar to each other, then are all squares similar to each other?*" Using the definition of similarity provided above, explain whether all squares are similar. Use complete sentences and diagrams if necessary.

3. Suppose the figure ABC is mapped onto the figure $A'B'C'$. Determine if the two objects are similar. Do not attempt to plot the figures. The coordinates for each figure are provided.

   $ABC \rightarrow\ A(q, w); B(r, w); C(s, t)$
   $A'B'C' \rightarrow A'(2q + 3, 2w); B'(2r + 3, 2w); C'(2s + 3, 2t)$