Chapter 5 – Congruent Triangles

5.1 Angles of Triangles
- 5.1.1 Triangle classification (sides and angles)
- 5.1.2 Triangle angle relationships (sum, exterior angles)

5.2 Congruent Polygons
- 5.2.1 Identifying and using corresponding parts of congruent figures
- 5.2.2 Proving triangles are congruent using SASASA

5.3 Proving Triangles Congruent by SAS
- 5.3.1 Formal proofs using previous properties (parallel lines, vertical, circles, etc)
- 5.3.2 Compass and straight edge copy of a triangle using SAS

5.4 Equilateral and Isosceles Triangles
- 5.4.1 Base angles of an isosceles triangle
- 5.4.2 Solve for unknown values in diagrams involving equilateral and isosceles triangles

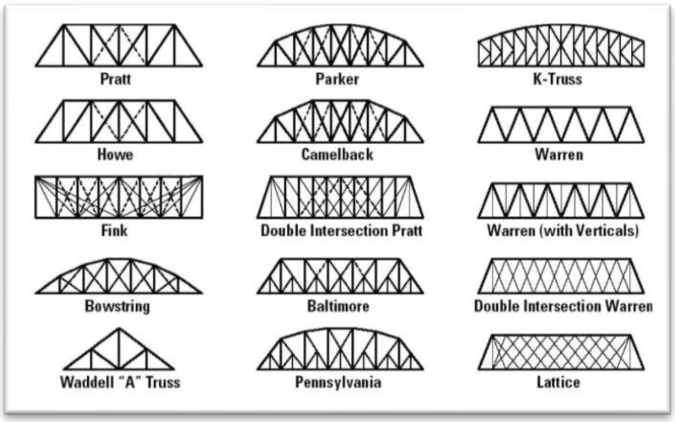
5.5 Proving Triangles Congruent by SSS
- 5.5.1 Formal proofs using previous properties (parallel lines, vertical, circles, etc)
- 5.5.2 When does SSA work? (HL theorem)
- 5.5.3 Compass and straight edge copy of a triangle using SSS

5.6 Proving Triangles Congruent by ASA and AAS
- 5.6.1 Formal proofs using previous properties (parallel lines, vertical, circles, etc)
- 5.6.2 Summarize congruent theorems (how to tell them apart)
- 5.6.3 Compass and straight edge copy of a triangle using ASA

5.7 Using Congruent Triangles
- 5.7.1 Indirect measurement
- 5.7.2 Proving validity of compass and straight-edge constructions

5.8 Coordinate Proofs
- 5.8.1 Distance and midpoint formula with variable coordinates

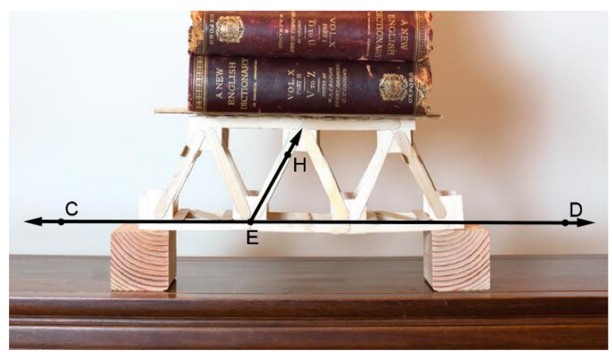
Practice Performance Task – 5.1
Bridges



A truss is a beam used to support a structure such as a roof or bridge. The common truss patterns used for bridges are shown to the right. Use your knowledge of triangle classification and angle relationships to answer the following questions.

1. The following image shows a bridge that utilizes one of the common truss patterns shown in the given diagram. Which pattern is used? Classify two triangle types used within the bridge. Outline them and explain how you made your judgement. (Explain why it is the type of the triangle you chose.)

2. The following image shows a Warren truss bridge. Suppose the interior triangles of the bridge are equilateral. What would be the measure of $\angle CEH$? Use complete sentences to explain how you determined this measure.



3. Suppose the same design was used (a Warren truss bridge) but it was decided that adding verticals would increase the strength of the bridge (see original diagram for example). Draw a sketch of the bridge from problem 2 with verticals added. Classify the new triangles formed and calculate the measure of each angle.

Practice Performance Task – 5.2
Gate Construction

The following gate uses intersecting boards that form a series of triangles to prevent sag and lean. It can be assumed that points that appear to be collinear are collinear.

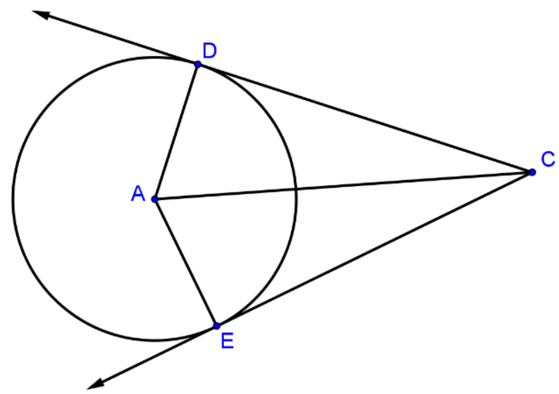
1. Suppose the directions for construction state that the three horizontal boards must be equally spaced. Is this sufficient information to prove both ΔCED and ΔFGE have three congruent angles? If so, complete a two-column proof. If not, explain which angles may differ.

2. If we think of ΔCDE as a reflection of ΔFGE over $\overline{JH}$, then list the corresponding pairs of sides in ΔCDE and ΔFGE which must be congruent.

3. The construction directions call for board EG and FG to form a 56° angle at point G. Provide reasons for each of the following conclusions in the following table. Assume all horizontal boards are parallel and all vertical boards are parallel.

Statements	**Reasons**
$\angle EGF = 56°$	Given
$\angle DCE = 56°$	
$\angle HGE = 34°$	
$\angle EHG = 90°$	
$\angle HEG = 34°$	

Practice Performance Task – 5.3
Jumping ahead



During the second semester, we will study the relationship between circles and several different kinds of lines. For example, $\overline{CD}$ and $\overline{CE}$ are called tangent lines because they barely touch the outside of the circle. Use what we have learned about congruent triangles to answer the following questions.

1. The two triangles shown in the diagram above (ΔDAC and ΔEAC) are formed using the two tangent lines. Which pairs of corresponding parts do you know are congruent? List the pairs below and use complete sentences to explain how you know they must be congruent.

2. Given the information above, is there enough information to prove the two triangles are congruent by SAS? Support your answer using complete sentences.

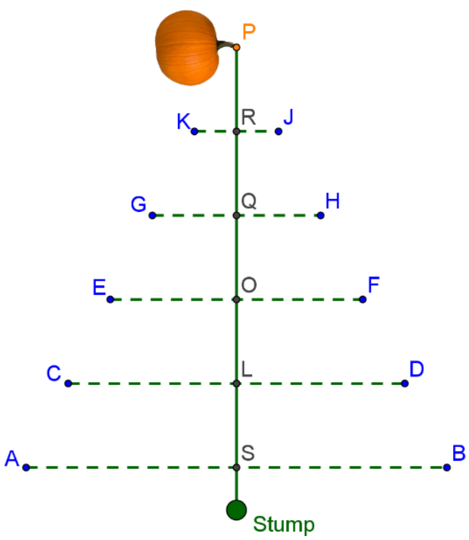
3. Suppose you were told $\angle DAC \cong \angle EAC$. Complete the two column proof below to prove $\Delta DAC \cong \Delta EAC$ using SAS.

Statements	Reasons

Practice Performance Task – 5.4
Pruning Giant Pumpkins

Growing giant pumpkins is a fun hobby that requires tons of planning, care and luck. In 2014, a man from Germany grew a 2,323-pound pumpkin (shown to the right)! The problems that follow will discuss the best way to trim pumpkin vines if your goal is to grow a giant.

The best way to trim the vines for giant pumpkins is using a Christmas tree pattern. The vines closest to the stump are allowed to grow the longest. As the vines approach the pumpkin, you trim them shorter. The diagram below shows this pattern.



1. Given only the information from the diagram and the above description, explain why we cannot prove ΔPAB forms an isosceles triangle. What additional information is necessary to prove ΔPAB is isosceles?

2. Suppose an expert grower has posted some useful information on his Facebook page. Using this information, find each of the following measurements.

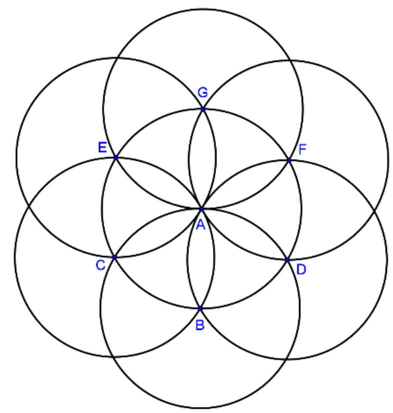
$m\angle PSB =$ $\quad\quad$ $m\angle KPR =$

$m\angle PAS =$ $\quad\quad$ $m\angle PBA =$

3. Again using the information from the Facebook post, complete the following proof that $\angle PAB \cong \angle PCD$.

Statements	Reasons

Practice Performance Task – 5.5
Flower of Life

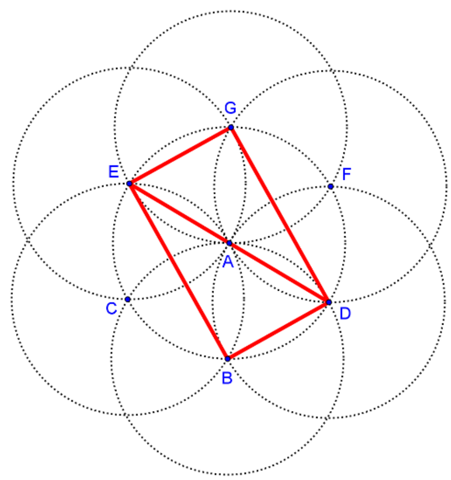


When I was in 5th grade I was shown how to make the following design using only a compass. A year later I won a design competition in which I integrated it into a logo. The figure is called The Flower of Life and consists of seven **congruent circles**. Use your knowledge of circles to answer the following questions concerning triangles present within the figure.

1. On the figure to the right, draw ΔAEG and ΔAFG. Use the table below to prove $\Delta AEG \cong \Delta AFG$.

Statements	Reasons

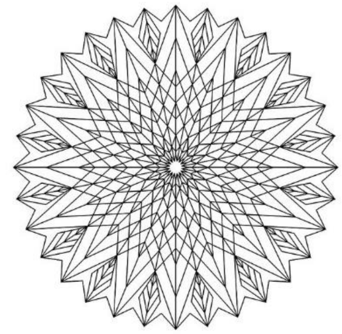
2. The diagram below shows two different triangles within circle A. Two separate, incomplete, proofs are shown. Complete each proof to show $\Delta EDG \cong \Delta DBE$. You may NOT add lines to either proof.



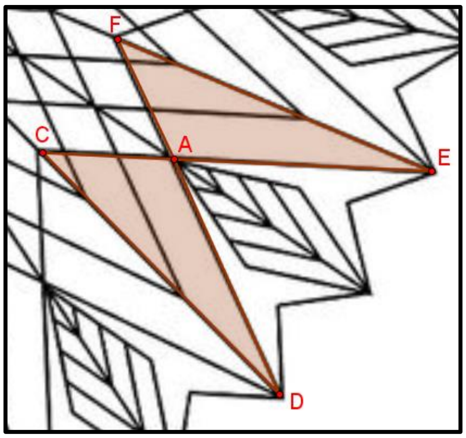
Statements	Reasons
$\overline{ED} \cong \overline{DE}$	
$\overline{EG} \cong \overline{DB}$	
	Double radii of congruent circles are congruent
$\Delta EDG \cong \Delta DBE$	

Statements	Reasons
$\angle EGD$ and $\angle DBE$ are 90°	Inscribed Δ with diameter as one side are right Δ
$\Delta EDG \cong \Delta DBE$	

Practice Performance Task – 5.6
Adult Coloring Books



The last couple of years, intricate coloring books aimed at teenagers and adults have become increasingly popular. The designs in these coloring books are typically more intricate than traditional coloring books for children. Geometric designs involving intersecting lines, polygons and circles are popular. The following problem focuses on congruent triangles present within the design to the right.



1. The image to the left shows an enlarged portion of the overall design. While there are no additional marks or given information, what corresponding parts of ΔCAD and ΔFAE must be congruent. Explain using complete sentences.

2. Suppose a teacher wants to make a triangle congruence problem using the above triangles. Sketch the two triangles shown above in the area below. Include enough "given" information by making appropriate marks on the diagram. The given information should allow a student to prove $\Delta CAD \cong \Delta FAE$ by AAS. Do not include more information than necessary. Afterwards, complete the two-column proof using your "given" information.

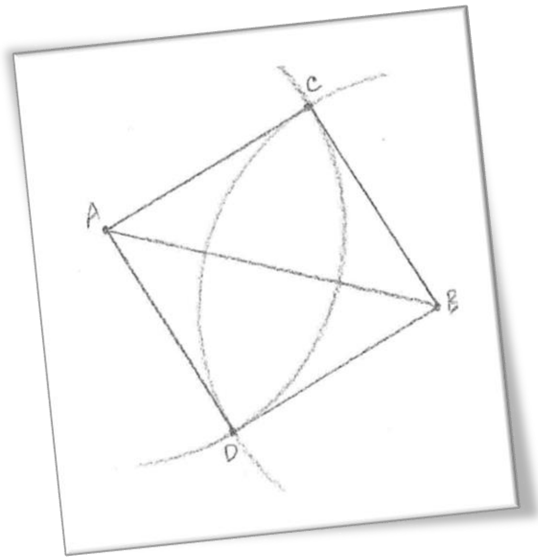
3. Suppose we were told $CD \parallel FE$. While they certainly do not look parallel, would this enough information to prove $\Delta CAD \cong \Delta FAE$? Use complete sentences in your explanation.

Practice Performance Task – 5.7
Compass and Straight-edge Constructions

While a straight-edge simply connects two points with a straight line, combined with a compass, one can draw congruent segments and angles. Use your knowledge of constructions and triangle congruence to determine if the reasoning behind each construction is valid.

The following construction was completed by a student who explained the steps who took:

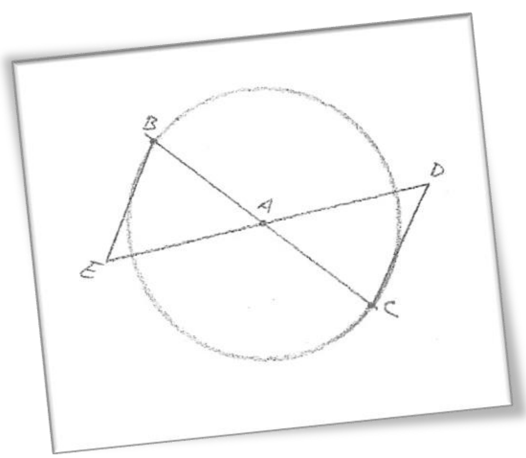
I started out with line segment AB. Then I opened my compass to over half AB and drew a half circle with A and then B as the center. I labeled the intersections of those half circles C and D. I connected A to C and D and then connected B to C and D.



1. Using the construction and the description provided, can we prove $\Delta ABC \cong \Delta ABD$? If so, explain how using any acceptable form of proof. If not, explain what corrections could be made to ensure the two triangles are congruent.

Once again, the following construction was completed and described by a student.

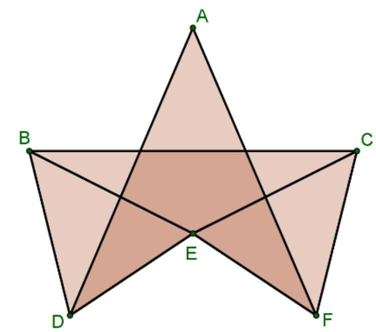
I first drew two intersection lines. I labeled the intersection A. Then I put the compass on A and drew a circle that intersected one of the lines two times. I called those intersections B and C. I labeled the ends of the other line segment D and E. Lastly, I connected B to E and C to D.



2. Using the construction and the description provided, can we prove $\Delta ABE \cong \Delta ACD$? If so, explain how using any acceptable form of proof. If not, explain what corrections could be made to ensure the two triangles are congruent.

Practice Performance Task – 5.8
Vector Graphics

Have you ever attempted to make a picture you took with your phone larger in order to make a poster or background image for your computer? You will notice that these images become blurry. Unlike bitmap-like images from your phone, vector graphics use polygons that assign rules concerning the appearance of an image. If you zoom in on a vector graphic, no such blurring will occur. Many businesses will create a vector graphic for their logo so that it does not appear blurry when put on large banners.



Consider the simple business logo shown to the right. Suppose one size of the image has the following coordinates for each point:

$A(0,6)$	$B(-4,3)$	$C(4,3)$
$D(-3,-1)$	$E(0,1)$	$F(3,-1)$

1. Prove ΔAFD forms an isosceles triangle.

2. Prove $\overline{AE}$ bisects $\overline{BC}$.

The vector version of this logo has the following coordinates for any value k.

$A(0,k)$	$B\left(-\frac{2k}{3},\frac{k}{2}\right)$	$C\left(\frac{2k}{3},\frac{k}{2}\right)$
$D\left(-\frac{k}{2},-\frac{k}{6}\right)$	$E\left(0,\frac{k}{6}\right)$	$F\left(\frac{k}{2},-\frac{k}{6}\right)$

3. Prove that for any value of k, ΔBEC is an isosceles triangle.