Chapter 6 – Relationships Within Triangles

6.1 Perpendicular and Angle Bisectors
- 6.1.1 Developing connection between perpendicular bisectors and isosceles triangles
- 6.1.2 Angle bisectors and the creation of congruent, right triangles
- 6.1.3 Finding equations for bisectors

6.2 Bisectors of Triangles
- 6.2.1 Circumcenter – Equidistant from each vertex. (Construction: perp. bisec. of each side)
- 6.2.2 Incenter – Equidistant from each side (Construction: bisect each angle)
- 6.2.3 Algebraic equalities/equations

6.3 Medians and Altitudes in Triangles
- 6.3.1 Centroid – Intersection of all medians (Construction: MP to vertex) (balancing point)
- 6.3.2 Orthocenter – Intersection of Altitudes

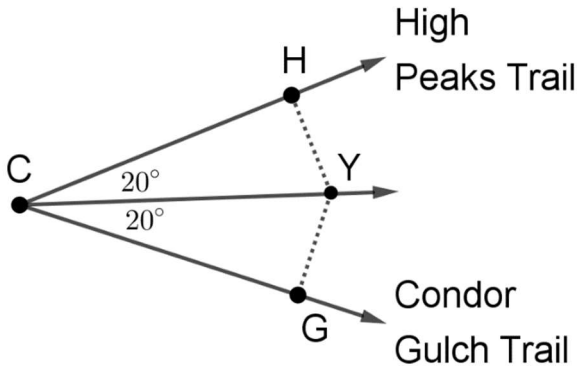
6.4 The Triangle Midsegment Theorem
- 6.4.1 Major properties of midsegments
- 6.4.2 Properties of a midsegment triangle

6.5 Indirect Proof and Inequalities in One Triangle
- 6.5.1 Constructing a proof/argument by contradiction
- 6.5.2 Larger side/angle and Triangle Inequality Theorems

6.6 Inequalities in Two Triangles
- 6.6.1 Hinge Theorem/converse (Larger side/angle but w/ 2 triangles)

Practice Performance Task – 6.1
Missing Person

You and your friend Brandon are camping where the High Peaks Trail and the Condor Gulch Trail intersect (C). Two hours ago, Brandon said he was going to go collect firewood, but he has not returned. You (Y) decide to begin searching for him by walking along a path that forms a 20° angle with each trail. The diagram to the right shows the path you take. The dotted lines represent your distance from each trail.



1. Solve for each of the missing angles in the two triangles.

   a. $m\angle CHY =$

   b. $m\angle CYH =$

   c. $m\angle CGY =$

   d. $m\angle CYG =$

2. As you walk along your path, looking for Brandon, are you closer to the High Peaks Trail, the Condor Gulch Trail, or are you the same distance from each? Support your answer using what you have learned in class.

3. Suppose you have walked 50 feet along your path and you are 17 feet from the high peeks trail. If you decide to walk straight to the high peaks trail and head back to camp (forget Brandon), then how far would you have to walk?

4. After 10 minutes of searching, you have walked a distance of 200 feet from camp (C) to where you are now (Y). You are 68.4 feet from each trail. Find the area of the region you have searched. (The area of CHYG. (Hint: $A_{\Delta} = \frac{b \cdot h}{2}$)

Practice Performance Task – 6.2
Is it possible?

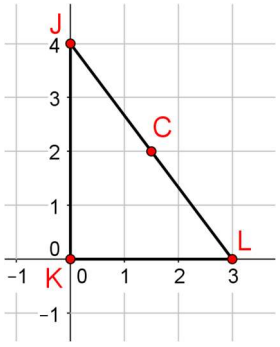
The circumcenter of a circle is the point which is equidistant from each of vertex. The incenter is the point which is equidistant from each side. The following questions are meant to challenge your understanding of each of these ideas by encouraging you to think creatively. Good luck!

1. Is it possible for the incenter of a circle to be the midpoint of one of the sides of the triangle as well? If yes, draw one such triangle. If no, explain why this would be impossible.

2. Is it possible for the circumcenter of a circle to be outside the triangle? If yes, then draw an example and explain what kinds of triangles have an exterior circumcenter. If no, then explain why this is impossible.

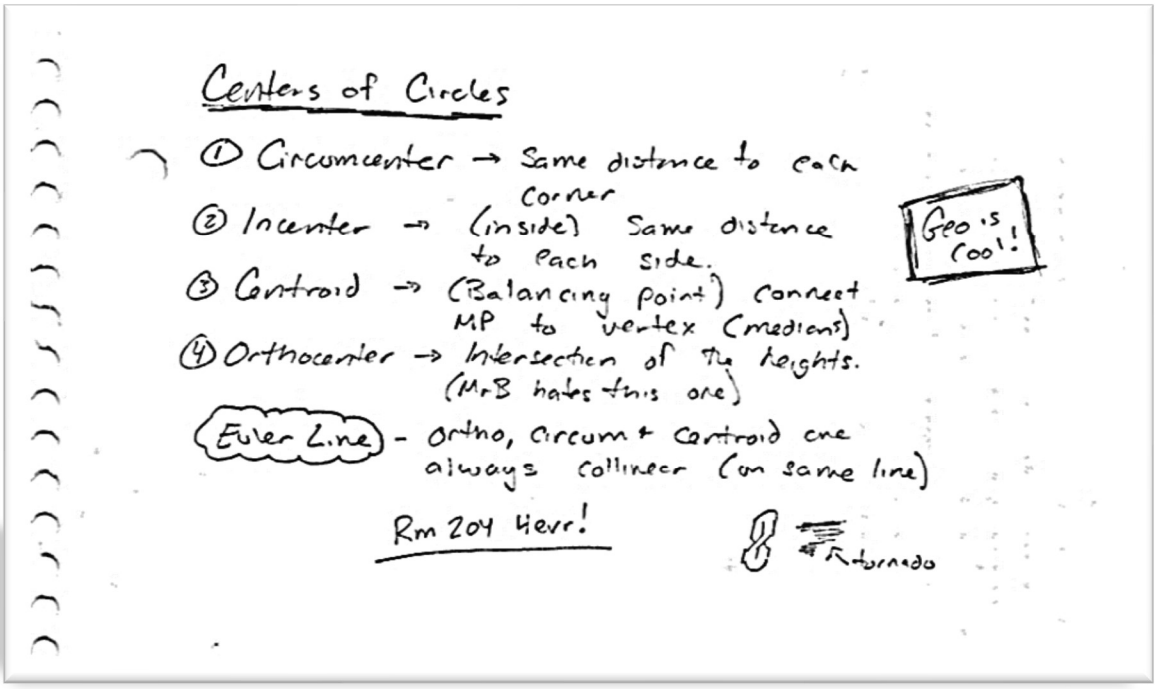
3. Is it possible for a triangle to have a circumcenter and incenter that are in the exact same location? If yes, draw one such triangle and show both the incenter and circumcenter are in the same location. If no, explain why this would be impossible.

4. One interesting property of circumcenters is that for right triangles, the circumcenter is always at the midpoint of the one of the sides. In this example, C is the circumcenter of ΔJKL. Find the area of the circle that circumscribes ΔJKL. (Remember $A_{circle} = \pi \cdot r^2$)



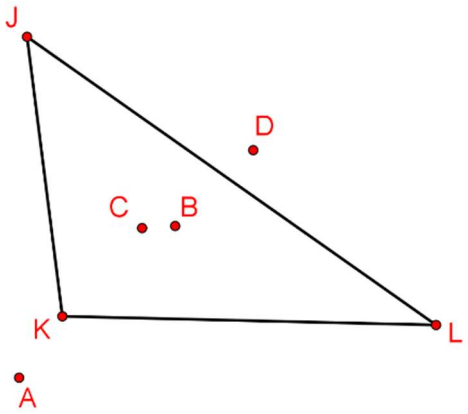
Practice Performance Task – 6.3
So Many Centers

Suppose you were absent for yesterday's lesson about the many centers of triangles. The bad news is there is a pop-quiz today about that lesson and you must take it. The good news is the teacher will let you use the notes your friend took during class. Use your friends notes to complete the following pop-quiz.



POP QUIZ!

The following image has each of the centers marked. For each center, explain which point represents that center and how you made your decision. You must explain your reasoning.



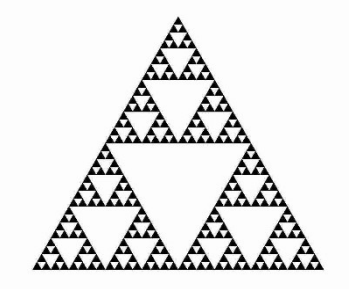
Which point is the Circumcenter?

Which point is the Incenter?

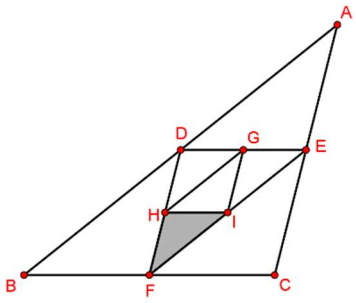
Which point is the Centroid?

Which point is the Orthocenter

Practice Performance Task – 6.4
Sierpinski-esque Triangle



The Sierpinski Triangle is a fractal (repetitive) design formed by repeatedly drawing midsegment triangles within an equilateral triangle. The figure to the right is the typical Sierpinski Triangle. The following problem explores a Sierpinski-esque Triangle formed using an obtuse triangle.



1. The figure to the left shows a Sierpinski-esque Triangle formed using an obtuse triangle (ΔABC). If the area of the shaded triangle (ΔHIF) is 2 in^2, then what is the area of the ΔABC?

2. Suppose the following side lengths are given: $DF = 4.2$, $DE = 4.0$, $EF = 6.4$. Find the perimeter of ΔGHI.

3. In the given diagram, $m\angle A = 37°$ and $m\angle ADE = 39°$. Use complete sentences to explain how to find $m\angle C$.

4. Suppose you are told the area of the trapezoid $HGEF$ is 15 in^2, explain how you could find the area of ΔABC. This question is separate from question 1-3. Ignore those answer when answering this question.

Practice Performance Task – 6.5
Popsicle Sticks

Your math teacher has a giant bucket of popsicle-sticks of different lengths. The shortest length is 1 inches and he has every whole number inch length up to 6 inches.

1. You draw three popsicle-sticks from the bucket. The first popsicle-stick is 1 inch long. Is it possible that when you draw your next two sticks, it will be impossible to make a triangle? Explain.

2. Again, assume the first stick you draw was 1 inch. Place an X in each cell that represents a situation in which you cannot make a triangle. For Example: If your second stick is 2-inches and your third is 6 inches, then you could not make a triangle. The "X" has been placed below.

	Length of 2nd popsicle-stick						
		1	2	3	4	5	6
Length of 3rd popsicle-stick	1						
	2						
	3						
	4						
	5						
	6		X				

3. If your first stick is 1 inch long, what is the probability you will be able to make a triangle after selecting your next two sticks?

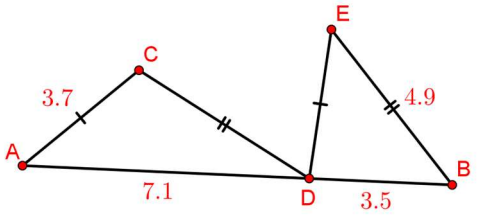
4. Is there any length you could draw for your first popsicle-stick that would guarantee you will be able to make a triangle after drawing your next two sticks?

5. Given the popsicle sticks in the bucket, how many different isosceles triangles are possible to construct?

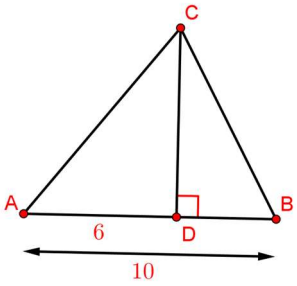
Practice Performance Task – 6.6
Why is that?

For each of the following diagrams, a student has written a conclusion. If you believe the conclusion is correct, then you must provide an argument in support (proof). If you believe the conclusion is incorrect, explain the nature of the error.

Statement #1: The measure of angle C must be greater than angle E.



Statement #2: The measure of angle CDA must be less than the measure of angle EDB.



Statement #3: Angle A and angle B are congruent.

Statement #4: The measure of ACD is greater than the measure of angle DCB.

Statement # 5: Line segment ED is congruent to line segment FG.

Statement #6: Line segment CB is twice as long as line segment FG.