Chapter 7 – Quadrilaterals and Other Polygons

7.1 Angles of Polygons

7.1.1 The sum of the interior angles of a polygon (triangle derivation and formula)

7.1.2 The sum of the exterior angles is always 360°

7.2 Properties of Parallelograms

7.2.1 Four major properties of a parallelogram:

a) Opposite sides are congruent
b) Opposite angles are congruent
c) Consecutive angles are supplementary
d) Diagonals bisect each other

7.2.2 Geometric congruence, algebraic equality, coordinate proof

7.3 Proving a Quadrilateral is a Parallelogram

7.3.1 Applying the converse to theorems from 7.2, prove a given quadrilateral is a parallelogram

7.3.2 Only non-converse theorem is one pair of sides being both congruent and parallel

7.4 Properties of Special Parallelograms

7.4.1 Rhombus (additional properties)

a) Diagonals are perpendicular
b) Diagonals bisect the angles

7.4.2 Rectangle (additional properties)

a) Diagonals are congruent

7.4.3 Square (additional properties)

a) Diagonals are perpendicular
b) Diagonals bisect the angles
c) Diagonal are congruent

7.5 Properties of Trapezoids and Kites

7.5.1 Isosceles Trapezoid (additional properties)

a) Base angles are congruent
b) Diagonal are congruent

7.5.2 Midsegments of trapezoids (average of base lengths)

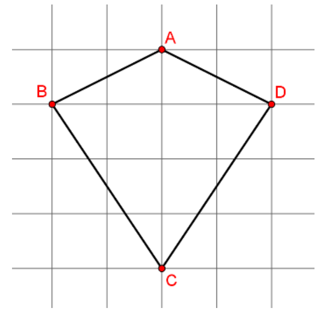
7.5.3 Kites (properties)

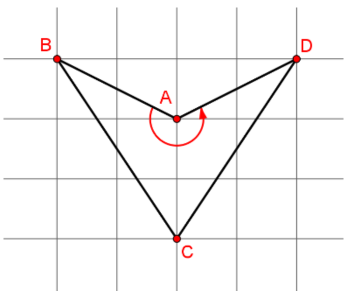
a) Diagonals are perpendicular
b) One pair of opposite angles are congruent

Practice Performance Task – 7.1
Concave Polygons

In high school geometry, we spend most our time studying convex polygons (a polygon with all interior angles less than 180°. The following questions investigate the properties of concave polygons (polygons with at least 1 interior angle greater than 180°.

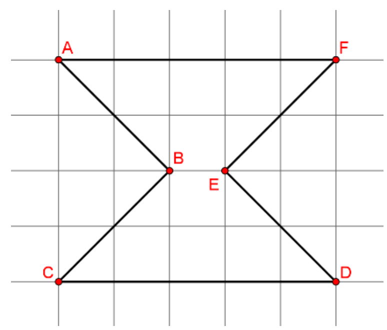
1. The figure to the right is a convex kite. $m\angle A = 127°$ and $m\angle C = 67°$. $\angle B \cong \angle D$. Find the measure of $\angle B$ and $\angle D$.

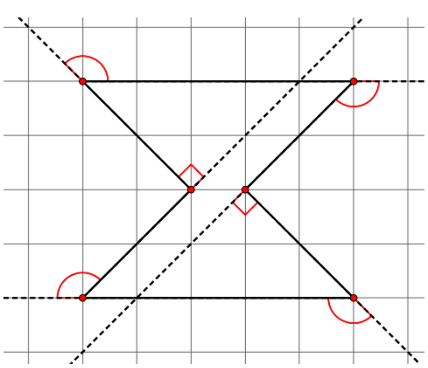




2. To the left, you have a concave kite, also known as a dart. The Polygon Interior Angle Theorem still applies to concave polygons. Once again, $\angle B \cong \angle D$. Using the measurements given in part 1, find the measure of each angle of the dart.

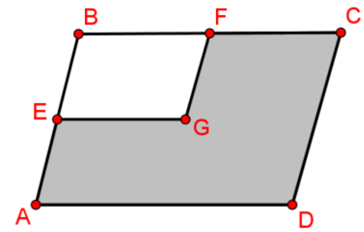
3. The figure to the right is a concave hexagon. Per the Polygon Interior Angle Theorem, what should be the sum of the interior angles? Find the measure of each interior angle, do they add up to the correct sum?





4. The figure to the right shows the exterior angles for the concave hexagon shown in problem 3. Per the Polygon Exterior Angle Theorem, the sum of these angle should be 360°. Find the sum of these angles. It may seem these angles do not add up to 360°, but they do. Lost yet? What's going on here?

Practice Performance Task – 7.2
Gnomons



In Euclid's Elements, Euclid refers to the *complement of a parallelogram* as the space left over when a smaller, similar parallelogram is removed from one of the corners. In modern geometry, this is called the *gnomon* of a parallelogram. The shaded region to the right would be a gnomon.

1. In the diagram above, $\square EBFG$ and $\square BADC$ are both parallelograms. How can we prove $\angle EAD \cong \angle BEG$?

2. Explain how we know $\angle G \cong \angle D$. (Hint: There a few ways to do this, some tricky, some not so tricky and some REALLY easy.)

3. Suppose F is the midpoint between B and C; E is the midpoint between A and B. If the area of the gnomon is 15, what would be the area of $\square ABCD$? Show your work.

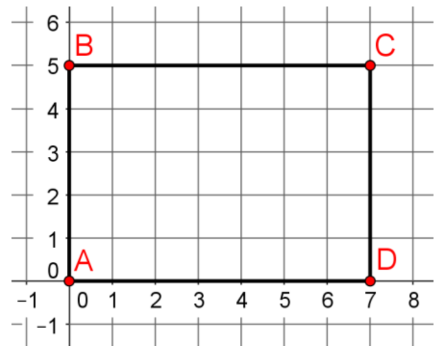
4. Once again, assume the F and E are midpoints of $\overline{BC}$ and $\overline{AB}$ respectively. Explain how we know G is the midpoint of the diagonal BD.

5. More generally, a gnomon can be defined as the space left over when a smaller, similar shape is removed from the larger shape. Assuming we begin with a parallelogram, what would the gnomon of the gnomon look like? (Good luck.)

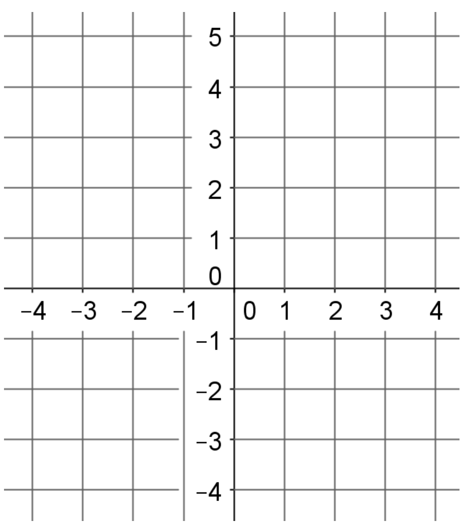
Practice Performance Task – 7.3
Inscribed Parallelograms

An interesting result in geometry is that connecting the midpoints of the sides of any quadrilateral will result in a parallelogram. The following performance task will investigate this result and some interesting corollaries.

1. On the grid to the right, rectangle ABCD has been plotted. Plot the midpoint for each side and then connect them to form a new quadrilateral, ▱EFGH.

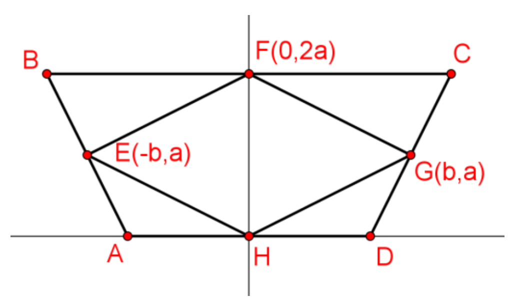


2. Prove ▱EFGH is a parallelogram by showing both pairs of opposite sides are congruent.



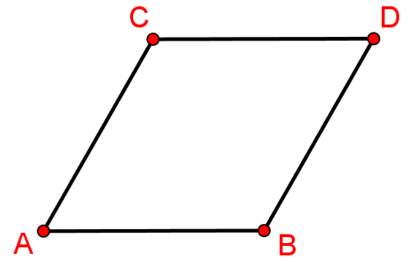
3. A student in your class claims that if you connect the midpoints of the sides of a kite, the new quadrilateral will not only be a parallelogram but it will also be a rectangle. Use the grid to the left to construct a kite of your choice, plot the midpoints and determine if the resulting figure is a rectangle. Explain your results using complete sentences.

4. The diagram to the right shows an isosceles trapezoid with the midpoints of its sides connected. The coordinates are given in *general*, meaning variables are used instead of exact values. Prove ▱EFGH is a parallelogram by showing its diagonals bisect.



Practice Performance Task – 7.4
The Bressette Rhombus

In my never-ending quest to invent something new in geometry, I have come up with a special kind of rhombus. The figure to the right is what I will call a "Bressette Rhombus". What makes the Bressette Rhombus special is that one of its diagonals is congruent to one of its sides.

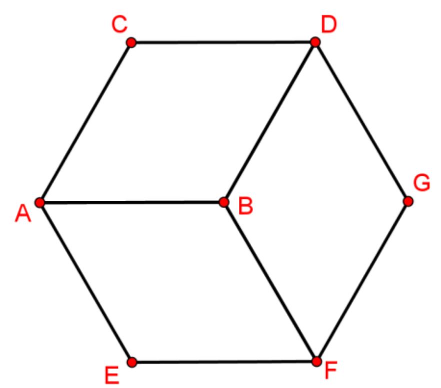


1. ▭ABDC is a Bressette Rhombus with the diagonal $\overline{CB}$ congruent to $\overline{AB}$. Draw $\overline{CB}$. What kind of triangle is ΔACB? Explain why $\Delta ACB \cong \Delta DCB$.

2. In a Bressette Rhombus, what is the measure of each obtuse angle? Explain how you found your answer.

3. Find the area of a Bressette Rhombus with diagonal lengths of 4 inches and 6.9 inches. Remember $A_\Delta = \frac{b \cdot h}{2}$.

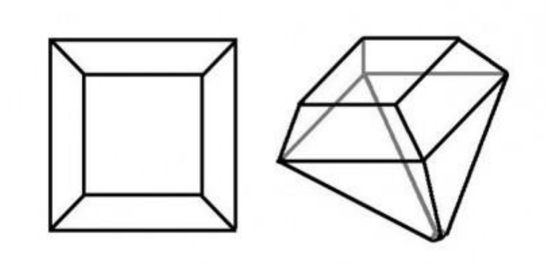
4. I claim the Bressette Rhombus is the only rhombus that can form a cube-like image using 3 congruent copies of itself. Is this true? Explain.

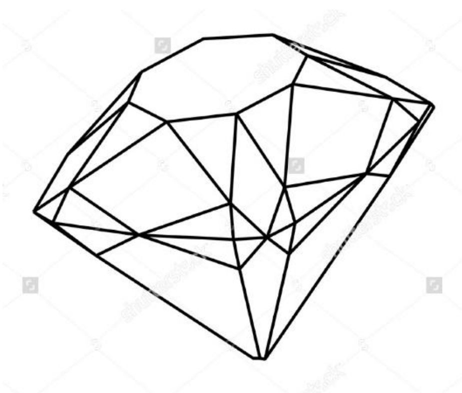


Practice Performance Task – 7.5
Diamonds

There are several different ways in which a diamond can be shaped. Each cut uses symmetry and geometric figures to create an elegant look that reflects the maximum amount of light back to the viewer.

1. The figure to the right shows a basic Table Cut shape which was very common during the 1300's. Using the figure to right, describe the quadrilaterals and triangles most likely used in a Table Cut. Use complete sentences.





2. In the 1700's, the Peruzzi Cut became popular. The diagram to the right shows a Peruzzi Cut. Describe the polygons present in a Peruzzi Cut. You may label the figure to aid in your description.

3. A two-dimensional version of the Peruzzi Cut is shown to the right. Using your description from part two along with the properties of special polygons, find the measures of angles 1, 2 and 3.