Chapter 9 – Right Triangles and Trigonometry

9.1 The Pythagorean Theorem

9.1.1 The basics of the Pythagorean Theorem and the existence of Pythagorean triples (simplifying radicals)

9.1.2 The Pythagorean Inequalities Theorems: Using the PT to classify triangles

9.2 Special Right Triangles

9.2.1 The 45/45/90 special right triangle as half of a square to explore properties

9.2.2 Generalizing with the formula: $hyp = SL \cdot \sqrt{2}$

9.2.3 The 30/60/29 special right triangle as half of an equilateral triangle to explore properties

9.2.4 Generalizing with the formulas: $hyp = 2 \cdot SL$ and $LL = SL \cdot \sqrt{3}$

9.3 Similar Right Triangles (nested)

9.3.1 Identifying, redrawing and labeling the three right triangles formed when the altitude to the hypotenuse is constructed

9.3.2 Existence and definition of the geometric mean.

9.3.3 Quadrature constructions using the geometric mean

9.4 The Tangent Ratio

9.4.1 Defining the tangent ratio using it to solve for missing side lengths

9.4.2 Using special right triangles to solve tangents of 30/45/60

9.5 The Sine and Cosine Ratios

9.5.1 Defining the sine and cosine ratios using it to solve for missing side lengths

9.5.2 Using SRT to solve sine and cosine of 30/45/60

9.5.3 Exploring the complementary relationship between sine and cosine

9.6 Inverse Trigonometric Ratios

9.6.1 Using the inverse trig ratios to solve for missing angles in right triangles

9.6.2 Solving for unknown angles of elevation/depression (angles off horizontal)

9.7 Law of Sines and Cosines

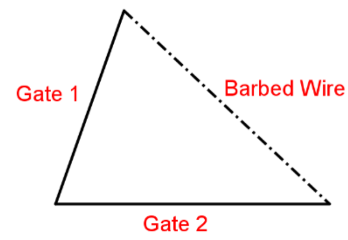
9.7.1 Using the trig ratios to find the area of non-right triangles

9.7.2 Law of Sines (looking for the known angle/side pair)

9.7.3 Law of Cosines (SSS and SAS)

Practice Performance Task 9.1
Fence Construction

A farmer wants to build a small pen for a year-old calf to get some sun and graze on the grass. He has two metal gates which he will use for two sides of his triangular pen. The farmer will use barbed-wire to as the third side to close the triangle. The diagram to the right gives you a rough idea of what the farmer is planning.



Gate 1 is 6 feet long and gate 2 is 8 feet long. The farmer has tons of barbed wire and therefore can make the third side any length he wants.

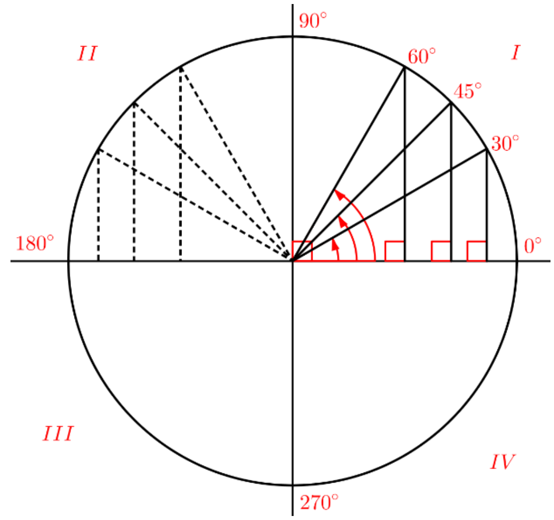
1. Suppose the farmer creates a pen for his cow using 11.5 feet of barbed wire. Describe the shape of the pen. Be specific.

2. What is the largest amount barbed wire the farmer could possibly use to create this pen? Explain how you got your answer. Do you think this pen would work well for the cow?

3. Suppose the farmer believes the largest possible pen requires the two gates to connect at a right angle. Describe how the farmer can easily construct the pen so that the two gates form a right angle. (He doesn't have a big protractor.) What is the area of his pen? (Remember: $A_\Delta = \frac{b \cdot h}{2}$)

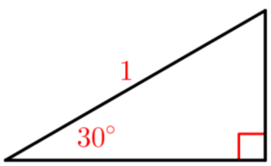
4. Do you believe the farmer is correct in assuming the largest possible triangle in this situation will result in the two gates forming a right angle? If you agree, explain how you could prove it. If you disagree, create a triangular pen with a larger area than the one the farmer created.

Practice Performance Task – 9.2
The Unit Circle



Special Right Triangles can be frustrating for students because they're not really used for much until later courses like Algebra II and Trigonometry. The diagram to the right shows what is called the Unit Circle. As you rotate around the circle, you start at 0° and go counter clockwise.

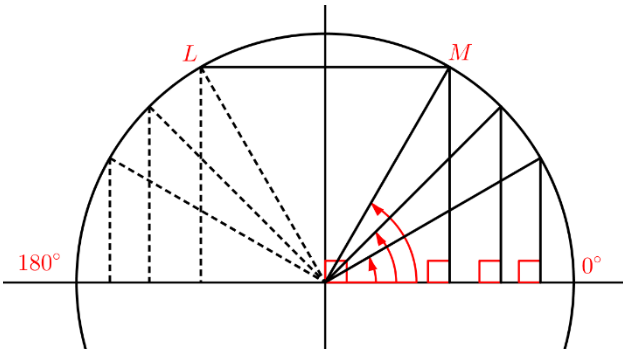
Quadrant I (from 0° to 90°) contains three special right triangles.



1. The radius of the Unit Circle is 1. The first triangle of the unit circle is shown to the left. Find the length of each of its sides.

2. The special right triangles for quadrant II are marked with dashed lines. Fill out the missing angle measurements for each of the triangles, like how it was done in quadrant I. (Hint: each of your angles must be between 90° and 180°.

3. Using your ruler, complete the bottom half of the Unit Circle. Make sure to mark each of the angles and draw all remaining triangles.

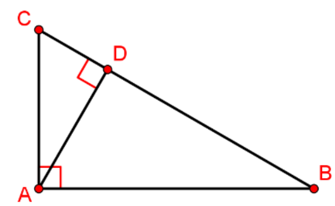


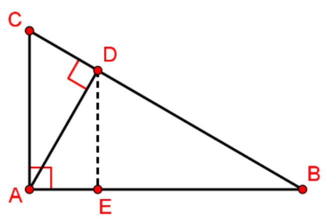
4. For the heck of it, I connected the top vertex from quadrant I and quadrant II. If we assume the radius of the unit circle is 1, then how long is line segment LM?

Practice Performance Task – 9.3
Fractal Means

A fractal is a geometric design created by repeating a pattern. The following problem will consider a fractal created using nested, right triangles like those shown in the diagram to the right.

1. Per the Geometric Mean Theorem, $\overline{AD}$ is the geometric mean of $\overline{CD}$ and $\overline{DB}$. Suppose $CD = 2$ and $DB = 7$, find the length of AD which represents the geometric mean of 2 and 7. (You may solve this problem geometrically or algebraically.)



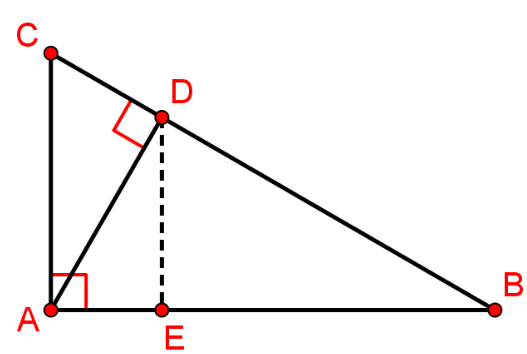


2. To start our fractal design, we will take the larger of the two triangles ΔADB and draw the altitude to the hypotenuse. The diagram to the left shows this next step. The new segment $\overline{DE}$ is the geometric mean of what two line segments? (Name the two segments, do not calculate their lengths.)

3. Using the figure from question 2, is there enough evidence to prove $\overline{CA}||\overline{DE}$? If yes, then explain how. If no, explain what information is missing.

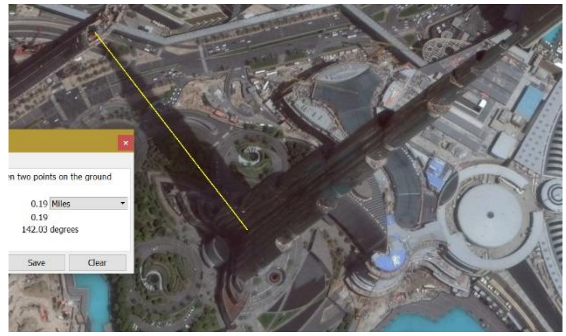
4. Using the figure to the right, draw the next two line segments to continue the fractal, label them $\overline{EF}$ and $\overline{FG}$.

5. $\overline{FG}$ is the geometric mean of what to segment lengths?



Practice Performance Task – 9.4
Shadows

Google Earth allows regular citizens to view extremely detailed satellite images from anywhere in the world. While these images are 2-dimensional, the heights of objects located in the images can be calculated using sun angles and shadows. Recently, a new class of North Korean submarine was tested for potential nuclear threat using shadows to determine the height of its tower. Let's try out this method on some known objects to see how well it works.



1. The image to the right shows the world's tallest building, the Burj Khalifa in Dubai, U.A.E. At the time this image was taken (about 11:00AM local time) the angle of elevation to the sun was approximately 70°. Using the ruler tool in Google Earth, the length of the shadow is .19 miles or about 1003 feet. Draw a diagram of this situation below and calculate the height of the Burj Khalifa.

2. The actual height of the Burj Khalifa is 2717 feet. Does your approximation based on shadow analysis seem reasonable? Explain.

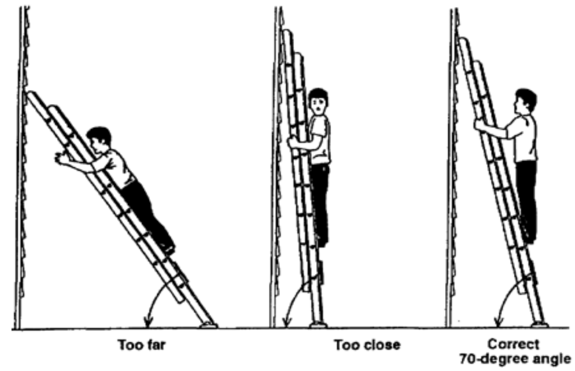
3. The following satellite image shows the new class of submarines recently (2014) discovered at a known naval base in North Korea. Suppose the sun's angle of elevation at the time the image was captured was 32.7° and the length of the shadow is approximately 70 feet. Estimate the height of the submarine's tower. (I don't know these values, I made them up.)

4. Suppose a sub must have a tower height of 30 feet to launch a nuclear missile. (Again, I don't know these values. I'm making them up.) Based on satellite images, is this sub capable of launching a nuclear missile?

Practice Performance Task – 9.5
Ladder Safety

Suppose there are plans to paint a large mural high on the wall of a building on campus. The painter is going to use a long ladder leaned up against the wall. Use this information to solve each of the following problems.

1. According to the diagram, the proper angle of elevation for operating a ladder is 70°. If the mural is 30 feet above the ground, then how long of a ladder must the painter use? Draw a diagram and show your work.



2. The painter will use a 31-foot ladder to reach the mural 30 feet above the ground. How many feet from the wall is the bottom of the ladder? Draw a diagram and show your work.

3. The ladder is 40 feet long and forms a 60° angle with the ground. Draw a diagram and solve for the height the ladder can reach and the distance from the wall to the bottom of the ladder.

4. A homeowner has purchased an extendable ladder from hardware store. The ladder can extend from a minimum length of 8 feet to a maximum of 15 feet. Find the range of reachable heights she can attain using a safe operating angle of 70°.

Practice Performance Task – 9.6
Pool Construction/Safety

The State of California has an extremely high number of safety regulations involved in the construction of a pool. The diagram to the right describes the restrictions concerning a ledge or step located under water.

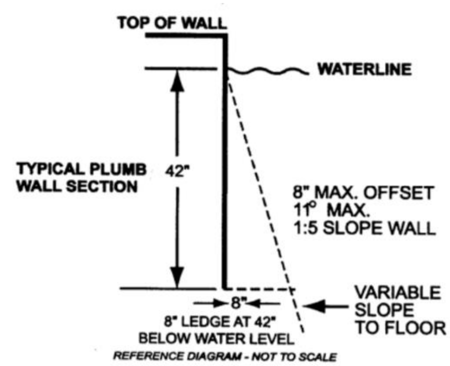
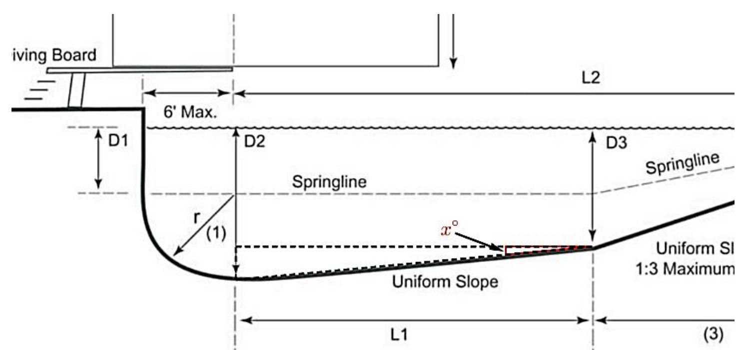


Figure 4- Offset ledges

1. The diagram describes the maximum allowable "ledge at a depth of 42". To the side, it says this corresponds to an 11° angle. Is this accurate? Setup an appropriate diagram using the 42" and 8" dimensions and solve for the maximum angle.

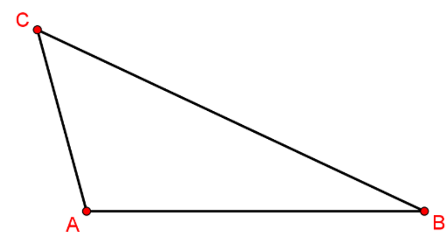
2. Before purchasing a home, an inspector checks the pool to make sure it meets the given requirements. In the deep-end of the pool, the ledge is 40" below the surface. If the ledge is 6" wide, does the pool meet the 11° maximum angle requirement described in figure 4? Setup a diagram and show your work.

3. The following diagram outlines required dimensions for pools with a 3-meter diving board. $D2 = 13'$, $D3 = 12'$, $L1 = 20'$. Determine angle of depression from the bottom of D3 to the bottom of D2.



Practice Performance Task – 9.7
Comparing Methods

There are two common methods for calculating the area of a non-right triangle. The first method we discussed in class involves using trigonometry to find the height. The second method, we have not yet discussed, is an intriguing result called Heron's Formula. You will be determining the area of a triangle using both methods and then comparing your answers.



1. ΔABC is shown to the right. The following is given: $AC = 5$, $AB = 9$, $BC = 11.38$ and $m\angle A = 105.25°$. Label the given diagram with all the given information.

2. Use trigonometry to calculate the height of ΔABC and then find its area. (Hint: extend BA as its base and draw the height to C.)

3. Heron's Formula can also be used to determine the area of ΔABC without using trigonometry. To use the formula, you must first determine the semi-perimeter which is half of the triangle's perimeter. Use the formula $S = \frac{a+b+c}{2}$ to determine the semi-perimeter of ΔABC.

4. Heron's Formula is as follows: $A = \sqrt{S(S-a)(S-b)(S-c)}$ where S is the semi-perimeter of the triangle. Use Heron's Formula to calculate the area of ΔABC. Does your answer match your answer in question 2? (It should…)

5. Solve the triangle by finding the measure of $\angle B$and $\angle C$.